

Selling with Binding Reservations in the Presence of Strategic Consumers

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We analyze a revenue management problem in which a seller endowed with an initial inventory operates a *selling with binding reservations* scheme. Upon arrival, each consumer, trying to maximize his own utility, must decide either to buy at the full price and get the item immediately or to place a nonwithdrawable reservation at a discount price and wait until the end of the sales season where the leftover units are allocated according to first-come-first-serve priority. We prove the existence of an equilibrium consumer's strategy in this game and develop a simple and accurate asymptotic approximation for it.

Through an extensive numerical study, we find that our proposed mechanism delivers higher revenues than the markdown practice with a preannounced fixed discount. The benefit is more emphasized when the seller is more patient than the consumers and (1) the ratio between the number of units put up for sale and the expected demand is moderate and/or (2) the heterogeneity of the consumers' valuations is moderate to high. In our numerical experiments, the revenue gap can reach more than 12%, which is quite significant for retail businesses that typically operate with narrow margins.

Key words: revenue management; retail operations; markdowns; strategic consumer behavior; Bayesian-Nash equilibrium; asymptotic analysis

History: Received April 6, 2008; accepted July 29, 2010, by Martin Lariviere, operations and supply chain management. Published online in *Articles in Advance* October 25, 2010.

1. Introduction

The use of markdowns for selling a limited supply of perishable assets is an extended practice in retailing. In the apparel industry, a recent study suggests that around 50% of the inventory of a typical retailer is sold at discount prices (e.g., see Hardman et al. 2007). Here, as well as in other comparable retail operations with high gross margins and low net profits, small changes in revenue can have a big impact on the financial performance.¹ Indeed, scientific pricing is considered the fastest and most cost-effective way to increase profits (see Phillips 2005, §1.2). Analysts and vendors tout a 1% to 3% boost in overall sales, and in some cases a 10% rise in gross margins for companies that employ price-optimization technology (e.g., see Sullivan 2005).

In this paper, we touch upon markdown pricing. This practice is suitable for settings where typically price insensitive customers arrive early in the sales

season and price sensitive customers arrive late, as it is the case for apparel, high-tech, and perishable-foods retailing and concert and sport events.² Its extended implementation has also raised some concerns among retailers, because consumers have been trained to strategize over the timing of their purchases and buy on sale. This higher market sophistication requires a refinement of the usual markdown practice that preserves the advantage of price discrimination but mitigates the downside of the consumers' strategic wait. One possible approach is the deliberate introduction of scarcity in the market. A well-known case is Zara, a Spanish apparel chain that sells 85% of its inventory at the regular price as opposed to 65% of its European peers (see Ghemawat and Nueno 2006). Zara sets low inventory levels for its fashion-sensitive products to induce consumers to buy early rather than wait for sales. The focus of our paper is the analysis of an alternative refinement of the markdown practice: the use of binding reservations under first-come-first-served, or first-in-first-out (FIFO) rationing rule.

¹ According to a report from Reuters dated February 5, 2008, the average gross margin across 63 apparel retailers based in the United States was 37.8% and the net profit was 6.6%. By the tighter economic times of April 2009, the average net profit across 119 "retail-apparel & accessories" companies was just 0.62%.

² Following the Desiraju and Shugan (1999) classification, these settings are labeled *class B* retailing services.

1.1. Motivation

We study a mechanism where consumers can place nonwithdrawable reservations for the leftover inventory that the retailer will clear at the end of the selling season. To our knowledge, the current use of this type of mechanism is very limited in the business practice.³ One of the few examples that we are aware of is the “plunging price” that was used online by Sam’s Club (<http://www.samsclub.com>) for a few years in the early 2000s to clear excess inventory. It consisted of a preannounced (price, time) schedule, where each consumer visiting the website could buy at the prevailing price or place a nonwithdrawable reservation through a credit card stand-by transaction at a specific future (price, time) combination. The reservation would be fulfilled provided there is a unit in stock by that time. Our model is a two-stage version of that (price, time) schedule.

Despite the discontinuity of the plunging price, we still think that Web-based business-to-consumer channels could become appropriate platforms to implement binding reservations. In fact, after the exponential growth of online auctions as an electronic retail selling mechanism (eBay being the canonical example), there has been a recent trend to meld them with more conventional fixed-price settings (for example, by designing auctions with “buy-now” options), or to even favor the use of plain fixed prices. An explanation for this shift is the proliferation of pricing information online that has made it easier for consumers to bargain hunt and lessened the need to risk overbidding in an auction (e.g., see Holahan 2008, Flynn 2008). While seeking ways of providing posted-price alternatives for consumers, online sellers are certainly still interested in mechanisms that allow price discrimination. The one that we explore in this paper follows this direction.

Binding reservations could also be applied in conventional bricks-and-mortar retailers through the installation of on-site kiosks so that consumers could choose a product using a graphical user interface and then swipe a credit card to put the purchase on hold. The reservation would become a stand-by transaction, reflecting the commitment to honor it provided that a unit is available at the end of the horizon, and the reservation time grants fulfillment under FIFO priority. By implementing this mechanism, the retailer would avoid the display of merchandise at low prices during a clearance season (which typically decreases the productivity of the shelf space), speed up the

introduction of new products, and reduce the holding cost incurred over the old merchandise. A downside could be given by the fact that availability of the reservation option might deter slightly compulsive consumers from buying immediately.

From the consumers’ perspective, our reservation mechanism is convenient because they would not need to revisit the store looking for a bargain. In addition, the FIFO priority rule delivers a sense of fairness because earlier reservations are honored first. Different from auctions, where allocations are theoretically founded on valuation-based priorities, the fairness here is rooted in time-based priorities that are observable firsthand. Moreover, our mechanism is easy to implement for the retailer and easy to explain to the consumers. It does not involve any a priori fee and reduces consumers’ search cost. Therefore, in principle, binding reservations exhibit desirable properties that, combined with a favorable revenue performance, would appeal to both consumers and retailers, providing a good support to pursue them in practice.

1.2. Overview of Main Results

In our model, a seller endowed with inventory of a particular product faces an arrival stream of consumers during a finite horizon. The seller announces the inventory put up for sale, the regular price p_h , and the clearance price p_l , with $p_l \leq p_h$. Arriving consumers must decide whether to buy at the full price p_h , or place a nonwithdrawable reservation and wait for the clearance season, where the leftover units (if any) will be allocated. Even though our approach allows for more general allocation rules, we concentrate the discussion on FIFO.

Given this nonwithdrawable reservation setting, how should strategic consumers behave? Certainly, consumers with valuations between p_l and p_h should place a reservation to get a nonnegative utility, but those consumers with valuation above p_h face a trade-off between getting a unit now at no risk by paying a high price, or placing a reservation with the hope of getting a unit at a low price later. Of course, the chance of getting a unit will depend on the purchasing behavior of other consumers. We prove that a symmetric purchasing equilibrium strategy for this noncooperative game exists, and that it is characterized by a threshold function in the space (time, valuation). The problem in this regard is its computational burden. To overcome this, we formulate an asymptotic version of the problem. We get a simple closed-form expression for the equilibrium strategy in this limiting regime, which is then used as an approximate solution for the original problem. Numerical computations show that this heuristic is very accurate for moderate- to large-size problems. Then, based on the limiting regime, we analyze the seller’s revenue optimization problem.

³ By July 2010, a binding reservation practice has been implemented by a commercial U.S. airline: The “AirTran U StandBy Ticket” program (<http://www.airtranu.com>), where college students can get nonconfirmed, deeply discounted tickets at the airports but could eventually need to wait at the gate for hours.

We analyze two benchmarks for our proposal: a fixed-price policy and a model of the markdown practice under preannounced discounts. The latter assumes a random allocation (RA) of units among consumers who revisit the store during the clearance season. Revenue-wise, FIFO dominates RA in general, which in turn dominates the fixed-price policy. The most beneficial scenarios in favor of our proposal occur when the seller's discount factor is lower than the consumers' and (1) there is a moderate supply with respect to the expected demand and/or (2) consumers are more heterogeneous with respect to valuations. The relative benefit of FIFO versus RA is even more emphasized when the clearance season is longer: Our numerical experiments show that their revenue gap can exceed 12%. The advantage of FIFO over RA is founded on the time-based, asymmetric criteria to allocate the excess supply of the regular season. We also verify that this advantage is still preserved when the market is composed by both myopic and strategic consumers.

1.3. Organization

The remainder of this paper is organized as follows: We review the related literature in §2. In §3, we introduce the model. The strategic behavior of consumers when facing the purchasing decision (i.e., "buy now" versus "place a reservation") is analyzed in §4. In particular, in §4.2 we describe one of our benchmarks: the regular markdown setting under a preannounced discount. The development of the asymptotic analysis for the mechanisms under consideration is included in §5, and the seller's revenue optimization problem is analyzed in §6. Our conclusions are summarized in §7. All the proofs are included in the e-companion.⁴

2. Literature Review

Recently, there has been a growing interest in the revenue management (RM) literature in capturing the intertemporal strategic behavior of consumers and developing ways to mitigate the adverse impact of this phenomenon on firms' revenue performance. A comprehensive reference on this topic is the book chapter by Aviv et al. (2009). A short list of the proposed mechanisms includes capacity rationing (e.g., Liu and van Ryzin 2008, Su 2007, Zhang and Cooper 2008), making price and capacity commitments (e.g., Elmaghraby et al. 2008, Aviv and Pazgal 2008, Su and Zhang 2008), using internal price matching policies (e.g., Levin et al. 2007), and limiting inventory information (e.g., Yin et al. 2009).

Clearly, the strategic behavior of consumers has challenged the pricing strategies of firms and inspired

the search of innovative mechanisms. Pricing with the added service of taking consumers' reservations is inscribed within this trend, and our paper contributes to it. In an earlier work, although in a setting where consumers' valuations and prices are likely to increase, Png (1989) shows that reservations are indeed very effective when risk-averse customers are uncertain about their own (future) valuations and available supply is limited. In his two-period model, if a customer decides to place a reservation in period 1, then he will exercise it in period 2 if the revealed valuation is high enough.⁵ A related pricing mechanism is given by what is named *contingent pricing* in the marketing literature, an agreement where a product is sold to a customer at a low price if the seller does not succeed in obtaining a higher price during a specified period. If a higher price is obtained during the arranged time period, the original sale does not take place, and the first potential buyer receives a previously agreed-upon compensation. The paper by Biyalogorsky and Gerstner (2004) analyzes this problem between a consumer and a seller for the single-unit supply case. The spirit of these models is to take advantage of an increasing pattern of prices, while securing advance sales at low prices.

More recently, reservations were studied in settings inspired by the restaurant business, though with modeling features different from ours (e.g., Alexandrov and Lariviere 2007 analyze withdrawable reservations, whereas Çil and Lariviere 2007 consider random rationing of scarce capacity).

DeGraba (1995) analyzes the convenience of selling to uninformed consumers early, before they learned about their valuations. He formulates a two-period model and argues that introducing scarcity in the first period induces the uninformed consumers to buy right away, because anyone waiting to purchase until becoming informed finds no units available. He describes this phenomena as "buying frenzy," which allows the seller to set a higher price, sell less units, and still get higher profits. His observation on that the firm can benefit from implementing an asymmetric rationing rule when demand exceeds supply is indeed related to our work.

A paper closer to ours is the one by Elmaghraby et al. (2009). They also study a problem where a (homogeneous) Poisson arriving stream of rational customers can choose either to buy during the selling season at a high price or to place a reservation at a preannounced low price and wait until the end of the season to get the product (subject to

⁴ An electronic companion to this paper is available as part of the online version that can be found at <http://mansci.journal.informs.org/>.

⁵ Png (1989) provides some rationality for why airlines take reservations free of charge and are willing to overbook. However, this reservations are withdrawable.

availability). They analyze the seller's expected payoff under this reservation regime and compare it with the no-reservation regime. They show that the seller prefers the reservation regime when there is a significant proportion of high-valuation consumers. Our work differs from theirs along several important modeling features. First, Elmaghraby et al. (2009) consider a single-unit supply case, whereas we consider a multiunit supply to be depleted during the sales horizon. This extension scales up the complexity of the analysis. Second, Elmaghraby et al. (2009) consider time-homogeneous valuations, whereas in our case consumers' valuations are time-nonhomogeneous and are discounted over the horizon. Therefore, in our case, when a consumer places a reservation, he does not only run the stockout risk, but also his payoff is discounted. This observation has a clear impact on the equilibrium behavior: Whereas in their case the equilibrium is defined by a single threshold time value, in our multiplicative utility function case, the equilibrium is defined by a continuous threshold function in the space (time, valuation). Third, Elmaghraby et al. (2009) assume that consumers are split in a finite number of segments, where segment i is defined by a constant valuation v_i . We assume that consumers are heterogeneous (in the sense that their valuations can differ) and that these valuations are private information taken from continuous probability distributions.

Another closely related paper is the aforementioned Aviv and Pazgal (2008). Our RA model follows from a setting introduced by these authors. They study two classes of pricing strategies for a single price-drop event: inventory-level-dependent and announced, fixed discounts. Through an extensive numerical study, they show that the latter could revenue-wise dominate the former by up to 8%, and they also estimate that the benefit of capturing explicitly strategic consumer behavior (versus ignoring it when customers are indeed strategic) could reach up to 20%. However, Aviv and Pazgal (2008) do not study the effect of different rationing rules. We take their generally dominant strategy (i.e., announced-fixed discounts) and use it as a benchmark for our proposed FIFO reservation mechanism. In addition, there are a couple of important technical differences between the two papers: First, we consider time-variant consumers' valuations; second, even though Aviv and Pazgal (2008) pose necessary conditions that a Bayesian-Nash equilibrium must satisfy, they do not demonstrate its existence.⁶

⁶ Aviv and Pazgal (2008), in Theorem 2 (p. 348), formulate the threshold function that would be the Bayesian-Nash equilibrium of the game under their announced fixed-discount pricing strategy. The proof of existence would require showing the convergence of the successive application of their equations (7) and (8), but they

A major technical contribution of our piece of research in this regard is proving the existence (and sufficient conditions for uniqueness) of an equilibrium under both FIFO and the random rationing rules, hence also reinforcing the support for the Aviv and Pazgal (2008) results.

The paper by Yin et al. (2009) is also related to ours; the authors present a model where consumers arriving during a selling horizon must decide to either buy immediately at a high price p_h or wait for a low price p_l that will be offered at the end of the season. Both prices are fixed and preannounced, and the leftover units are allocated randomly among the consumers who decided to wait. However, there are major differences with respect to our model. First, and most importantly, the focus of their paper is different from ours: They study the impact of two different inventory display formats and verify that by displaying one unit at a time (as opposed to all the available inventory), the seller is able to introduce a sense of scarcity in the market and achieve higher profits. We explore the use of binding reservations as a way to increase the retailer revenues. Second (time-homogeneous valuations) and third (two-customer segments with fixed valuations v_i) are similar to the setting in Elmaghraby et al. (2009). One advantage of Yin et al. (2009) is that consumers have real-time information of the inventory level. In our stochastic model, customers just know if the item is in-stock or sold-out, but the asymptotic analysis gives a first-order approximation for the inventory level in real time. Indeed, a distinguishing characteristic of our piece of research is the asymptotic analysis of the game, which provides simple and well-behaved heuristics for the rationing allocation rules that we study. Our approach in this regard follows the asymptotic analysis of Maglaras and Meissner (2006).

3. Model Description

A retailer (seller) is endowed with an initial inventory \bar{Q} of a homogeneous product. The inventory needs to be depleted over a selling season of length T . Following a RM approach, we assume that the inventory is not replenished. The seller can ration the inventory by choosing a quantity $Q_0 \leq \bar{Q}$ to put up for sale. The remaining quantity $\bar{Q} - Q_0$ is discarded at no extra cost or salvage value.

The seller announces a regular unit price p_h to be posted during the interval $[0, T]$ and a discounted price $p_l \leq p_h$ that will be realized at the end of the horizon. At time t , she also announces the time left

do not prove such a result. A related comment acknowledging this limitation appears at the end of their §4.3, where they claim that the proof of convergence of their iterative algorithm "presents an even harder theoretical challenge" (p. 347, footnote 9).

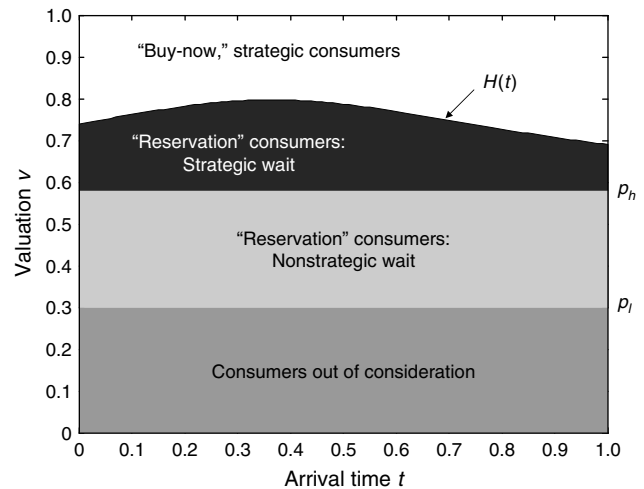
for the season end, $T - t$, and the inventory Q_0 initially available for sale. We assume that there is no real-time update for the current inventory level Q_t during $(0, T]$, but upon each arrival the seller discloses whether or not the product is available (i.e., she reveals if $Q_t > 0$ or not).

Both for the seller and for any arriving consumer during $[0, T]$, the number of units available at time T is described by a random variable Q_T , with support $\{0, \dots, Q_0\}$. The clearance is modeled as an instantaneous event that occurs right after time T , and where transactions at price p_l are due to consumer reservations placed during the regular season. If the number of reservations placed is less than or equal to the leftover inventory level, then all reservations are fulfilled. Otherwise, units are allocated following a rationing rule set in advance by the seller. In principle, we allow any time-based rationing rule that defines a total order among reservations with probability one (w.p.1) (details provided in §4), but focus our attention on a particular case: FIFO priorities.

On the demand side, the description of customer arrivals is similar to the one in Bitran and Mondschein (1997). Consumers have single-unit requests and visit the store or website following a nonhomogeneous Poisson process with intensity $\{\lambda(t): t \in [0, T]\}$. They are characterized by two quantities: (i) their arrival time and (ii) their private valuation for the product. For notational convenience, we denote the private valuation of a consumer arriving at time t by v_t . Observe that this notation is well defined because, w.p.1, the Poisson process has at most one arrival at any given time. The cumulative probability distribution F of the random variable v_t is allowed to be time dependent, to account for the dynamics of consumers' preferences (e.g., a valuation for a winter coat for a consumer in New York may be low in September, high in November, and then lower in February, near the end of the season). Let $F(v, t)$ have the common support $\mathcal{V} \triangleq [0, \bar{v}] \times [0, T]$, with $\bar{v} > p_h$. We assume that $F(v, t)$ is continuously differentiable in v for all t and admits a density function $f(v, t)$. Both λ and F are common knowledge. Without loss of generality, we assume from now on that $\bar{v} = 1$; that is, we scale all prices in this economy by \bar{v} .

When visiting the store, consumers must choose either to buy the product at the current price p_h or to reserve it for the later price p_l (and run the risk of not getting it), with the objective of maximizing their own surplus. We assume that they are sensitive to delay, and denote by $u(t, \tau, v - p)$ a quasi-linear discounted utility function of a consumer arriving at time t with valuation v who eventually gets at time τ a unit of product at price p (paid at the moment of

Figure 1 An Example of a Purchasing Strategy $H(t)$ Under the FIFO Rationing Rule for the Case $w = T = 1$, $Q_0 = 9$, $\lambda = 15$, $p_l = 0.3$, and $p_h = 0.58$, Where the Valuations Are Time-Homogeneous and Unif[0, 1]



Note. Consumers are classified into four categories according to their arrival time t and valuation v .

getting it). In particular, we consider an exponentially discounted utility function of the form:

$$u(t, \tau, v - p) = (v - p) \exp(-w(\tau - t)), \quad (1)$$

where w is a fixed, nonnegative constant shared by all consumers that captures their disutility for waiting.⁷

We assume that a consumer arriving at $t \in [0, T]$ bases his purchasing decision on his private valuation, his knowledge of the arrival rate $\{\lambda(s): s \in [0, T]\}$ and the distribution of valuations F , the initial inventory Q_0 , the remaining season time $T - t$, the announced prices, and the rationing rule for reservations. Pictorially, the consumers' type space (arrival time, valuation) is divided into four regions, as shown in Figure 1. Consumers with valuation below p_l quit without making any transaction. For those with valuation between p_l and p_h the only profitable option is to place a reservation. Consumers with valuation above p_h act strategically according to a threshold function $H(\cdot)$, such that a v_t -consumer reserves an item only if his valuation verifies $p_h < v_t \leq H(t)$. Those with valuation $v_t > H(t)$ are the *buy-now* consumers. We will characterize such $H(\cdot)$ in §4.

Figure 1 also provides an insight for the behavior of strategic consumers under FIFO. Among them, those with valuation slightly above p_h would tend to place reservations, because the tiny difference with

⁷Note that the consumer's utility function is of the intertemporal type (e.g., see Mas-Colell et al. 1995, Chap. 20). We assume the exponential decay due to technical convenience, because it guarantees nonnegativity for all t and τ , but our main theoretical results are not tight to this functional form of utility, as long as it remains increasing in $v - p$ and decreasing in delay $\tau - t$.

respect to p_h makes the instantaneous payoff small (and hence, the wait profitable). Those with very high valuations would tend to buy now in order to avoid the risk of waiting. For those with moderately high valuations, there could be two reasons for buying now: Certainly, one reason is to arrive late, when the chances of getting a unit from the FIFO rule are low. This is reflected by the decaying shape of $H(\cdot)$ toward the end of the horizon. But also, depending on the parameters of the problem, $H(\cdot)$ could be increasing at the beginning of the horizon (as is the case here), capturing the fact that the utility discount faced by early arrivals makes the wait less appealing.

The retailer’s problem is to design the selling season by setting the values for T , Q_0 , p_l , and p_h in order to maximize her expected revenue, which is also exponentially discounted over time. In this regard, the game between the seller and the consumers is of the Stackelberg-type, with the seller being the leader and the consumers being the followers.

4. Strategic Consumer’s Purchasing Behavior

In this section we study consumers’ purchasing decisions. We focus on the *strategic consumers*, i.e., those with valuation $v \geq p_h$. We assume that the seller has already announced the parameters of the selling mechanism (Q_0 , T , p_l , and p_h) and the rationing rule. The seller’s problem of optimally designing the mechanism is postponed to §6.

For ease of exposition, we rescale consumers’ valuations (and the corresponding probability distributions), price p_h , and arrival rate $\lambda(t)$, under the normalization $p_l = 0$. That is, based on the original value p_l , we set

$$\begin{aligned} v &\leftarrow \frac{v - p_l}{1 - p_l}, & p_h &\leftarrow \frac{p_h - p_l}{1 - p_l}, \\ \lambda(t) &\leftarrow \lambda(t)(1 - F(p_l, t)), \\ F(v, t) &\leftarrow \frac{F(v, t) - F(p_l, t)}{1 - F(p_l, t)}. \end{aligned} \tag{2}$$

Then, we set $p_l \leftarrow 0$. Note that under this scaling the range of valuations remains $[0, 1]$. If the consumers’ valuations are time heterogeneous, then this scaling results in a time-dependent arrival rate. We define the maximum (scaled) arrival intensity $\bar{\lambda} \triangleq \max_{t \in [0, T]} \lambda(t)$.

We can characterize the decision of a consumer arriving at time t with private valuation v_t by a threshold function $H(t)$ such that the consumer will place a reservation if and only if $v_t \leq H(t)$. The fact that we can represent the purchasing strategy for all t -consumers by a single threshold $H(t)$ is a consequence of the monotonicity of the utility function in the instantaneous payoff $v_t - p$. In other words, if it

is optimal for a v_t -consumer to wait $T - t$ time units for the clearance season then it is also optimal to wait for any other consumer arriving at t with valuation lower than v_t .

Two assumptions are used in this representation of the purchasing strategy. First, note that this characterization is based on the notion of a symmetric equilibrium in which all consumers use the same threshold function $H(t)$. In addition, we are also assuming that a consumer arriving at time t is incapable of observing the number of reservations placed and units left in the system. That is, we are assuming that the only information that a consumer uses to decide whether or not to place a reservation—besides λ , T , Q_0 , F , p_h , $p_l = 0$, and the rationing rule—is his arrival time and private valuation.

We will denote by \mathcal{H} the strategy space. To keep our formulation reasonably simple, we assume that $\mathcal{H} \subseteq \mathcal{D}$, the set of piecewise continuous functions with right and left limits, which is broad enough to include most strategies that are practically relevant. We will show that for a large class of rationing rules, the set \mathcal{D} is larger than necessary in the sense that in equilibrium any symmetric purchasing strategy $H \in \mathcal{H}$ is actually continuous. Note that by our scaling based on $\bar{v} = 1$ and $p_l = 0$, the elements of \mathcal{H} are functions with domain $[0, T]$ taking values in $[0, 1]$. Furthermore, for any $H \in \mathcal{H}$, we must have $H(t) \geq p_h$. This reflects the fact illustrated in Figure 1 that any consumer with valuation less than p_h cannot afford to buy the product during the regular season; the reservation is his only potentially profitable decision, no matter his arrival time. In summary, we define the set of potential purchasing strategies as the set of functions $\mathcal{H} = \{H \in \mathcal{D}, H: [0, T] \rightarrow [0, 1], \text{ such that } H(t) \geq p_h \text{ for all } t\}$.

We use a two-step approach to characterize a symmetric purchasing equilibrium (PE) $H \in \mathcal{H}$. First, we look at a consumer’s best-response purchasing strategy assuming that other consumers use a fixed strategy $H \in \mathcal{H}$. We will denote by $\mathcal{R}(H) \in \mathcal{H}$ this best-response purchasing strategy and refer to \mathcal{R} as the best-response mapping on \mathcal{H} . Second, we impose the equilibrium condition $\mathcal{R}(H^*) = H^*$.

Suppose a consumer—which we refer to as consumer τ —arrives at time τ with private valuation $v_\tau > p_h$, and suppose that every other consumer is using the purchasing strategy H . The relevant case for a consumer is when there are still units available upon his arrival (i.e., $Q_\tau > 0$).⁸ If consumer τ

⁸ Announcing the exact number of available units and reservations with a higher priority can simplify the consumer’s problem only if the number of those reservations exceeds the number of available units. This effectively leaves only the buy-now option available and consumers will behave myopically. If the number of reservations

decides to buy a unit at p_h , then his expected utility would be $u(\tau, \tau, v_\tau - p_h)$. On the other hand, if he decides to reserve, then his expected payoff would be $u(\tau, T, v_\tau - p_l)\mathbb{P}(\text{reserving at } \tau \text{ and getting an item at } T \mid Q_\tau > 0)$. Thus, a rational consumer τ places a reservation if and only if

$$u(\tau, T, v_\tau - p_l)\mathbb{P}(\text{reserve and get an item} \mid Q_\tau > 0, H) \geq u(\tau, \tau, v_\tau - p_h),$$

or equivalently, he places a reservation when

$$u(\tau, T, v_\tau - p_l)\Pi_H(\tau) \geq u(\tau, \tau, v_\tau - p_h)\mathbb{P}(Q_\tau > 0 \mid H), \quad (3)$$

where $\Pi_H(\tau)$ is the (unconditional) probability of getting an item through a reservation placed at time τ . We still need to explicitly characterize this reservation condition in terms of the function H , which will depend on the time-based rationing rule chosen.

The rationing rule is an ordering relationship governing priorities among placed reservations. It can be defined as a function $\xi(\tau): [0, T] \rightarrow [0, 1]$ that assigns a time-based priority to each reservation so that if $\xi(\tau_1) > \xi(\tau_2)$, then a customer arriving at time τ_1 has higher priority than one arriving at τ_2 in case both place reservations. The priority ordering is assumed to be strict, in the sense that w.p.1, for arrival times $\tau_1 \neq \tau_2$, either $\xi(\tau_1) > \xi(\tau_2)$ or $\xi(\tau_1) < \xi(\tau_2)$. For example, the FIFO rationing rule, i.e., “earlier reservations first,” is a strict ordering relationship and would correspond to a strictly decreasing function $\xi(\cdot)$. Assume $\xi \in \Xi \triangleq \{\xi \in \mathcal{D}, \xi: [0, T] \rightarrow [0, 1], \text{ such that the number of local extrema is finite}\}$. Note that $\xi(\cdot)$ is well defined for every $\tau \in [0, T]$, which guarantees that any possible reservation has an assigned priority. See §A2 of the e-companion for illustrations of different priority rules $\xi \in \Xi$.

Given $H \in \mathcal{H}$ and $\xi \in \Xi$, next quantities are important in the derivation of $\Pi_H(\tau)$. Define for all $\tau \in [0, T]$:

- Average number of consumers that “buy now” during $[0, \tau]$,

$$\Lambda_{H_B}(\tau) \triangleq \int_0^\tau \lambda(t)\bar{F}(H(t), t) dt, \quad (4)$$

where $\bar{F}(x, t) \triangleq 1 - F(x, t)$.

- Average number of consumers whose reservations have a higher priority than the reservations placed at time τ ,

$$\Lambda_{H_R}(\tau) \triangleq \int_0^T \mathbb{1}\{\xi(t) > \xi(\tau)\}\lambda(t)F(H(t), t) dt. \quad (5)$$

is smaller than the number of available units, consumers still need to assess the probability of getting a unit via reservations, and the decision problem will incorporate one more dimension.

Because $\mathcal{H} \subset \mathcal{D}$ and $\xi \in \Xi$, both $\Lambda_{H_B}(\tau)$ and $\Lambda_{H_R}(\tau)$ are well-defined functions, continuous in τ . Consider a customer who arrived at time τ and under strategy H placed a reservation with priority $\xi(\tau)$. Following our reasoning, the probability of the customer getting an item depends on the priorities of other customers’ reservations.

4.1. Strict Priority Rationing Rules and the FIFO Case

Let us denote by $N(x)$ a Poisson random variable with mean x . If, w.p.1, no two reservations can have the same priority, each customer can get an item through the reservation channel if the seller has available units after serving (i) all buy-now customers and (ii) all reservations with higher priorities. These quantities are independent, Poisson random variables $N(\Lambda_{H_B}(T))$ and $N(\Lambda_{H_R}(\tau))$, respectively, and the probability of getting an item through a reservation placed at time τ is given by

$$\Pi_H(\tau) = \mathbb{P}(N(\Lambda_{H_B}(T)) + N(\Lambda_{H_R}(\tau)) \leq Q_0 - 1). \quad (6)$$

Given $H \in \mathcal{H}$, we compute the best-response strategy $\mathcal{R}(H)$ for consumer τ by looking at the threshold function that is consistent with (3). For the exponentially discounted utility function defined in Equation (1), and according to condition (3), a consumer places a reservation when

$$\frac{v_\tau}{v_\tau - p_h} \geq \frac{\exp(w(T - \tau))\mathbb{P}(Q_\tau > 0 \mid H)}{\Pi_H(\tau)} \triangleq g_H(\tau). \quad (7)$$

Two important features of the elements of \mathcal{H} are also derived in §A1.2 of the e-companion:

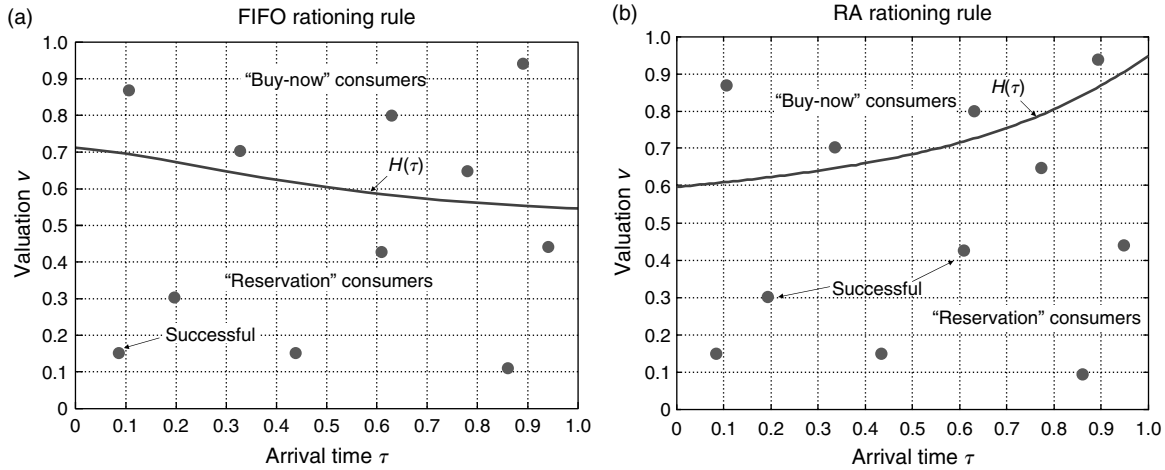
- There is always a range of consumers with valuations above p_h that prefer to reserve an item, irrespective of their arrival times (Lemma A4 in the e-companion).
- The best response strategy $\mathcal{R}(H)(\tau)$ is continuous in $[0, T]$; hence, \mathcal{R} effectively maps \mathcal{H} into \mathcal{H} . In particular, a consumer arriving at time τ with valuation v_τ places a reservation if and only if $v_\tau \leq \mathcal{R}(H)(\tau)$, where

$$\mathcal{R}(H)(\tau) \triangleq \begin{cases} 1 & \text{if } g_H(\tau) \leq \frac{1}{1 - p_h}, \\ \frac{p_h g_H(\tau)}{g_H(\tau) - 1} & \text{if } g_H(\tau) > \frac{1}{1 - p_h} \end{cases} \quad (8)$$

(see Proposition A1 in the e-companion).

A way to prove the existence of a symmetric equilibrium $H(\tau)$ is to verify that the best response mapping $\mathcal{R}(H)$ has a fixed point in the strategy set \mathcal{H} . This result is formally stated in Theorem A1 in the

Figure 2 An Example of a Purchasing Strategy $H(\tau)$ Under the FIFO (a) and Random Allocation (b) Rationing Rules with Parameters $w = 1$, $Q_0 = 6$, $\lambda = 10$, $p_h = 0.5$, $p_l = 0$, and Time-Homogeneous Valuations $\text{Unif}[0, 1]$



Notes. The same sample path of the arrival process (dots) is processed under both mechanisms. For FIFO, assuming $T = 1$, five consumers buy at the full price p_h and six place reservations. The earliest reservation gets the leftover unit. For RA, assuming $T_s = 1$, four consumers buy at the full price p_h , and seven place reservations, out of which two (randomly picked) get the leftover units.

e-companion. A less technical version of this result can be formulated as follows:

THEOREM 1. *For any rationing rule defined by $\xi(\cdot) \in \Xi$, there exists a symmetric equilibrium $H \in \mathcal{H}$. Moreover, under a stronger condition that depends on the parameters of the problem, the equilibrium is guaranteed to be unique and can be found through the iteration $H^{(n+1)} = \mathcal{R}(H^{(n)})$ starting from an arbitrary $H^{(0)} \in \mathcal{H}$.*

We refer the reader to Figure 2(a) for an example of equilibrium purchasing strategy under FIFO. As mentioned earlier, we will focus our discussion on the FIFO rationing rule to run comparisons with the usual markdown practice. We point out here that in all our numerical experiments under FIFO, we were always able to find a fixed point using the iterative procedure of Theorem 1.

4.2. Benchmark Setting: Clearance Season with Random Allocation

This benchmark model generalizes the announced fixed-discount setting presented in Aviv and Pazgal (2008, §5) to the time-nonhomogeneous valuation case. According to the numerical experiments there, this policy usually dominates the inventory-level-dependent pricing scheme in the presence of forward-looking consumers, and stands as challenging to beat. Aviv and Pazgal (2008) explain this dominance by arguing that a credible precommitment to a fixed-discount level removes the hope of consumers on deep discounts during the clearance season.

The sales horizon of length T is split in two periods: $[0, T_s]$, and $(T_s, T]$. During the first period, a price p_h is charged, and from time T_s onward, p_l is charged. All other basic details of our previous model also

apply here (i.e., Poisson arrivals at scaled rate $\lambda(t)$ during the whole horizon $[0, T]$, time-dependent valuations following distribution F , utility function as in (1)). The strategic behavior occurs during the first period, when arriving consumers must decide either to purchase at the current price p_h or come back at time T_s and buy at price p_l (subject to product availability). At time T_s , leftover units are allocated randomly among consumers who revisit the store.⁹ Consumers keep coming at rate $\lambda(t)$ during $(T_s, T]$. However, there is no gaming behavior of these late buyers; they just take the leftover inventory (if any).

We call this model the *random allocation* (RA) case, and think of the strategic-wait consumers as if they were placing reservations that have the same priority and will be honored randomly at time T_s according to a discrete uniform distribution. We stress here that the lottery is just modeling the allocation of the excess inventory among returning consumers; it is not being proposed as a reservation mechanism in the sense described earlier because it could be perceived as unfair. Under RA, the probability of getting an item for any strategic consumer who places a reservation is

$$c(H) \triangleq \min \left\{ \frac{Q_{T_s}}{\# \text{ of consumers that reserved an item}}, 1 \right\},$$

⁹ Note that T_s represents the moment of price reduction and instantaneous clearing of postponed purchases, similarly to the end of the horizon for strict priority rationing rules. According to a recent empirical study based on U.S. consumer data (Nakamura and Steinsson 2008), the median duration for a regular price season is between 7 and 9 months, with an average between 9 and 10 months. The average length of sales is between 1.8 and 2.3 months. This corresponds to $T_s/T \approx 0.8$ on average, with longer sales prevalent in apparel and household furnishings.

where the number of consumers who reserve an item is a Poisson random variable with mean

$$\Lambda_{H_R}^{RA}(T_S) = \int_0^{T_S} \lambda(t)F(H(t), t) dt. \quad (9)$$

Under RA, any strategic consumer who placed a reservation has the same probability of getting an item. In contrast, FIFO introduces an asymmetry among consumers, by giving a higher priority to earlier reservations. We can derive the condition for reservation from (3) and prove the existence of an equilibrium similarly to Theorem 1; see §A1.2.2 of the e-companion. Figure 2(b) illustrates an example of the RA rationing allocation rule for the same parameters as in the FIFO case in Figure 2(a).

5. Asymptotic Analysis of the Game

To circumvent the computational burden and lack of general convergence guarantee of the iterative procedure in Theorem 1, we present here a fluid model derived by replacing the stochastic demand by a continuous flow, with intensity set at the arrival rate. The analytical derivation relies on a limit of the model under a strong-law-of-large-numbers type of scaling when we let the demand rate $\lambda(t), t \in [0, T]$, and the initial number of units Q_0 grow proportionally large. The resulting limiting regime is equivalent to a model with deterministic demand. Indeed, a modeler may use our asymptotic approach as a direct way to tackle a deterministic model with no integrality constraints.

To derive the limiting model, consider a sequence of instances of the problem indexed by n so that $\lambda^{(n)}(t) \triangleq n\lambda(t)$ and $Q_0^{(n)} \triangleq nQ_0$ are the corresponding demand rate and initial inventory level for instance n , respectively, and let n increase to infinity. All other parameters are kept independent of n .

For each instance n of the problem, we let $\rho^{(n)} \triangleq Q_0^{(n)} / (\int_0^T \lambda^{(n)}(t) dt)$. Then, $\lim_{n \rightarrow \infty} \rho^{(n)} = \rho$, for $\rho \triangleq Q_0 / (\int_0^T \lambda(t) dt)$. We refer to ρ as the *supply-demand ratio*;

it represents the average number of units available per arriving consumer, and denote by $Q_\tau^{(n)}$ the random number of units available at time τ for instance n of the sequence of problems. In what follows, we analyze the limiting regime for the FIFO and RA rationing rules.

5.1. FIFO Rationing Rule

For the FIFO rationing rule, we use the related notation introduced in §4. The following result characterizes the asymptotic regime.

THEOREM 2. *Suppose that the purchasing strategy $H(\tau)$ is given. Then, in the limit as $n \rightarrow \infty$, consumers can deduce the following:*

(i) *The exact number of buy-nows, reservations, and available units, as they converge almost surely (a.s.), and uniformly in τ to their expected values:*

$$\begin{aligned} N(\Lambda_{H_B}^{(n)}(\tau))/n &\rightarrow \Lambda_{H_B}(\tau), \\ Q_\tau^{(n)}/n &\rightarrow Q_\tau \triangleq (Q_0 - \Lambda_{H_B}(\tau))^+, \quad \text{and} \\ N(\Lambda_{H_R}^{(n)}(\tau))/n &\rightarrow \Lambda_{H_R}(\tau). \end{aligned}$$

(ii) *The probability of getting an item through a reservation $\Pi_H^{(n)}(\tau) \triangleq \mathbb{P}(N(\Lambda_{H_R}^{(n)}(\tau)) \leq Q_\tau^{(n)} - 1)$ that converges to the two-point distribution:*

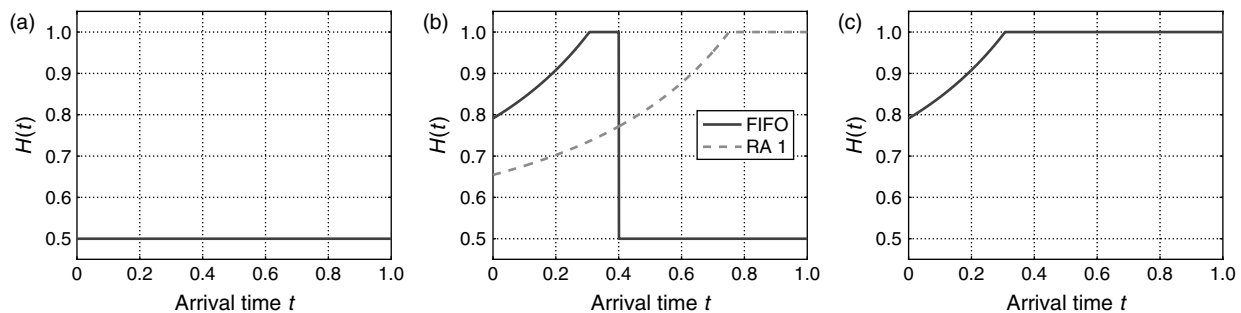
$$\Pi_H^\infty(\tau) = \begin{cases} 1 & \text{if } \Lambda_{H_R}(\tau) \leq Q_\tau, \\ 0 & \text{if } \Lambda_{H_R}(\tau) > Q_\tau \end{cases}$$

for $Q_\tau \triangleq (Q_0 - \Lambda_{H_B}(\tau))^+$.

Using the asymptotic approximation of Theorem 2, an equilibrium strategy can be computed through evaluation of the condition for reservation (3). Figure 3 illustrates three limit equilibrium strategies for the FIFO case depending on the supply demand ratio:

(i) **Limited supply.** When $Q_0 \leq \int_0^T \lambda(t) \bar{F}(p_h, t) dt$ the supply is limited, in the sense that there is not enough inventory to satisfy the demand from

Figure 3 Asymptotic Strategies Under FIFO and RA-1: (a) Limited Supply Case, $\rho = 0.4$; (b) Intermediate Supply Case, $\rho = 0.7$; (c) Abundant Supply Case, $\rho = 1$



Notes. Default values of parameters are $p_h = 0.5, T = T_S = 1, \lambda = 10, w = 1$, and time-homogeneous valuations $\text{Unif}[0, 1]$. Asymptotic strategies under FIFO and RA-1 coincide if the supply is abundant or limited.

consumers with valuations $v \geq p_h$. In this case the equilibrium strategy is to buy now if the valuation is greater or equal than p_h . Indeed, consider an arriving consumer v_τ , and suppose that all other consumers choose the strategy $H^*(t) = p_h, \forall t \in [0, T]$. Given that the supply scarcity ensures no leftover inventory at time T , the arriving player follows the strategy $H^*(\tau) = p_h$ (Figure 3(a)). We note that $H^*(\tau) = p_h$ is not the only PE in this case. In fact, let τ^* be a solution to $Q_0 = \int_0^{\tau^*} \lambda(t) \bar{F}(p_h, t) dt$. Note that τ^* is guaranteed to exist in the interval $[0, T]$ for the limited supply case. Then, any H of the form

$$H(\tau) = \begin{cases} p_h & \text{if } \tau \leq \tau^*, \\ s(\tau) \in [0, 1] & \text{if } \tau > \tau^*, \end{cases} \quad (10)$$

for any arbitrary function $s(\tau) \in [0, 1]$, is indeed an equilibrium, because for such an H , all Q_0 units will be depleted by time τ^* (i.e., any consumer arriving after τ^* will never get a unit, and so he becomes indifferent between placing or not a reservation).

(ii) **Intermediate supply.** If $\int_0^T \lambda(t) \bar{F}(p_h, t) dt < Q_0 < \int_0^T \lambda(t) dt$, all consumers with valuations $v \geq p_h$ can get an item, as well as some of the consumers with $p_l \leq v < p_h$. The consumers who obtain units through the buy-now or reservation channels are the ones with valuation $v \geq p_h$ plus the early arrivals up to time τ^* , which is the solution to

$$\int_0^{\tau^*} \lambda(t) dt + \int_{\tau^*}^T \lambda(t) \bar{F}(p_h, t) dt = Q_0. \quad (11)$$

Note that τ^* is guaranteed to exist in the interval $[0, T]$.¹⁰ From τ^* onward, only those with valuation $v > p_h$ get units from the buy-now channel. The early-arriving consumers, with $\tau \in [0, \tau^*)$, must decide which channel to purchase from, and therefore need to solve the limiting version of Equation (7):

$$\frac{v_\tau}{v_\tau - p_h} \geq \frac{\exp(w(T - \tau)) \mathbb{P}(Q_\tau > 0 | H)}{\Pi_H^\infty(\tau)},$$

where according to Theorem 2 and the selection of τ^* , both probabilities in the right-hand side are one. We conclude that in this *intermediate* case, the unique PE $H^*(\tau)$ is given by

$$H^*(\tau) = \begin{cases} \min \left\{ \frac{p_h \exp(w(T - \tau))}{\exp(w(T - \tau)) - 1}, 1 \right\} & \text{if } \tau \in [0, \tau^*], \\ p_h & \text{if } \tau \in (\tau^*, T], \end{cases}$$

for τ^* defined by (11) (Figure 3(b)).

¹⁰ For the time-homogeneous valuation case where $F(v, t) = F(v), \forall t$, Equation (11) admits a simple closed-form solution: $\tau^* = (Q_0 - \lambda T \bar{F}(p_h)) / (\lambda F(p_h))$.

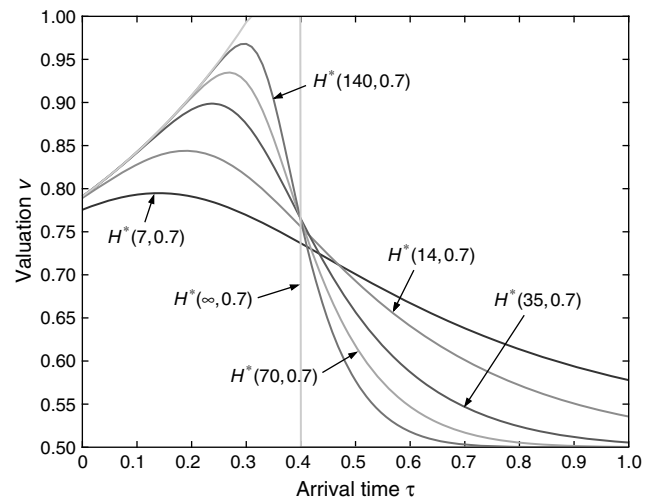
(iii) **Abundant supply.** If $Q_0 \geq \int_0^T \lambda(t) dt$ (i.e., $\rho \geq 1$), then every consumer will get an item from the channel he chooses w.p.1. The unique optimal strategy is given by

$$H^*(\tau) = \min \left\{ \frac{p_h \exp(w(T - \tau))}{\exp(w(T - \tau)) - 1}, 1 \right\}. \quad (12)$$

The result can be viewed as a particular instance of the intermediate supply case where the strategy $H^*(\tau) = p_h, \tau \in [\tau^*, T]$ is never realized because $\tau^* \geq T$. This case is illustrated in Figure 3(c).

With a slight abuse of notation, let $H^*(Q_0, \rho)$ be the optimal purchasing strategy if the seller offers Q_0 units and the supply-demand ratio is equal to ρ . In Figure 4 we compare the optimal asymptotic participation strategy $H^*(\infty, 0.7)$ (computed using Theorem 2) with several optimal purchasing strategies computed numerically using the iteration in Theorem 1. This scenario matches the intermediate supply case (case (ii) above). By setting $T = 1$, starting from $Q_0 = 7$ and $\lambda = 10$, we test the accuracy of the asymptotic approximation for systems with scale factors $n = 1, 2, 5, 10$, and 20, where a system of size n is defined by $\lambda^{(n)} = n \lambda$ and $Q_0^{(n)} = n Q_0$. The error

Figure 4 Performance of the Asymptotic Approximation for the FIFO Rationing Rule for an Intermediate Supply Case with Time-Homogeneous Valuations Unif[0, 1], $T = w = 1$, $\rho_h = 0.5$, and $\rho = 0.7$



Q_0	Exact PE		Approx. PE		Error (%)
	Reservations	Buy-nows	Reservations	Buy-nows	
7	6.98	3.02	6.56	3.44	8.40
14	13.93	6.07	13.13	6.87	8.08
35	34.26	15.74	32.82	17.18	5.79
70	67.73	32.27	65.63	34.37	4.21
140	134.60	65.40	131.26	68.74	3.34

Note. In this case, the exhausting time for the winning reservations is $\tau^* = 0.4$.

(last column in the table) is calculated as the fraction of consumers that make a suboptimal decision under the approximated PE, calculated as if all consumers play $H(\infty, \rho)$, with respect to the decision they would have made under the exact PE. We can observe that even for a moderate number of units, the approximation is satisfactorily accurate.

5.2. RA Rationing Rule

Let us consider the purchasing strategy under the RA rationing rule in the asymptotic regime. Recall that under the RA rule, the probability of getting an item is equal for all consumers who place a reservation.

THEOREM 3. *Suppose that the purchasing strategy $H(\tau)$ is given. Then, in the limit as $n \rightarrow \infty$, consumers can deduce the probability of getting an item after placing a reservation, as it converges to*

$$c^\infty(H) \triangleq \min \left\{ \frac{(Q_0 - \Lambda_{H_R}^{RA}(T_S))^+}{\Lambda_{H_R}^{RA}(T_S)}, 1 \right\} \text{ a.s.,} \quad (13)$$

where $\Lambda_{H_R}^{RA}(T_S) = \int_0^{T_S} \lambda(t) \bar{F}(H(t), t) dt$ and $\Lambda_{H_R}^{RA}(T_S)$ is defined in (9).

Again, we consider three different supply cases, which are illustrated on Figure 3. To ease exposition we reverse the order here:

(i) **Abundant supply** (i.e., $Q_0 \geq \int_0^{T_S} \lambda(t) dt$, Figure 3(c)). Here $c^\infty(H^*) = 1$, and the equilibrium strategy H^* is given by (12), with T replaced by T_S .

(ii) **Intermediate supply** (i.e., $\int_0^{T_S} \lambda(t) \bar{F}(p_h, t) dt < Q_0 < \int_0^{T_S} \lambda(t) dt$, Figure 3(b)). We can rewrite (13) as

$$c^\infty(H) = 1 - \frac{(1 - \rho) \int_0^{T_S} \lambda(t) dt}{\Lambda_{H_R}^{RA}(T_S)}, \quad (14)$$

where $0 < c^\infty(H) < 1$. The intermediate supply regime ensures that the items are available for buy-now purchases throughout the regular selling season, i.e., $\mathbb{P}(Q_\tau > 0 | H) = 1$ for all $\tau \in [0, T_S]$. Therefore, the optimal purchasing strategy is defined by the following threshold:

$$H^*(\tau) = \min \left\{ \frac{p_h \exp(w(T_S - \tau))}{\exp(w(T_S - \tau)) - c^\infty(H^*)}, 1 \right\}. \quad (15)$$

By substituting (15) into the denominator of (14), we get the equivalent condition

$$c^\infty(H^*) = 1 - \left((1 - \rho) \int_0^{T_S} \lambda(t) dt \right) \cdot \left(\int_0^{T_S} \lambda(t) F \left(\min \{ p_h \exp(w(T_S - t)) / (\exp(w(T_S - t)) - c^\infty(H^*)), 1 \}, t \right) dt \right)^{-1}. \quad (16)$$

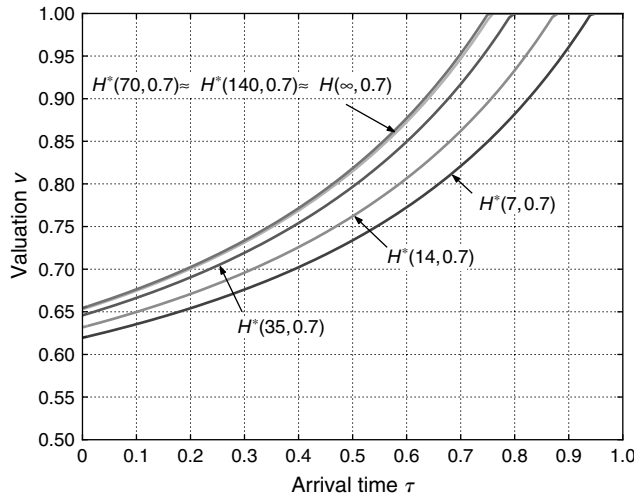
Equation (16) is a fixed-point equation in $c^\infty(H^*)$. Note that given $c^\infty(H^*)$, $H^*(\tau)$ is uniquely defined by (15). Proposition A4 in the e-companion shows that this fixed-point equation always has a solution for the intermediate supply case. Nevertheless, the uniqueness of the solution to (16) cannot be guaranteed, and in cases where there are multiple equilibria, it can be proved that the Pareto dominant equilibrium is the one with highest value $c^\infty(H)$ among the solutions to (16). This derivation is also included in the e-companion (§A1.3).

(iii) **Limited supply** (i.e., $Q_0 \leq \int_0^{T_S} \lambda(t) \bar{F}(p_h, t) dt$, Figure 3(a)). Multiple equilibria are also possible in this case. The purchasing strategy $H^*(\tau) = p_h$ for all $\tau \in [0, T_S]$, and more generally, any strategy given by (10) for $\tau \in [0, T_S]$ is an equilibrium. The probability of getting an item through a reservation is $c^\infty(H^*) = 0$ in this case. In addition, there could be another type of equilibria in which consumers can get an item through a reservation with $c^\infty(H^*) > 0$. In this scenario, an equilibrium is given by (15), where $c^\infty(H^*)$ is defined by (16). If a solution exists, it is not necessarily unique (see §A1.3 in the e-companion). Like in the intermediate supply case, if there are multiple equilibria, it could be verified that the one with highest value of $c^\infty(H^*)$ is Pareto-dominant.

We also compare here the optimal asymptotic purchasing strategy $H^*(\infty, 0.7)$ (computed from (16) and (15), for which the solution to (16) is unique) with several optimal stochastic purchasing strategies computed numerically using the iteration in Theorem A2 in the e-companion. Starting from $Q_0 = 7$, we test the accuracy of system scale factors $n = 1, 2, 5, 10$, and 20. Figure 5 shows the performance of the exact PEs with respect to the approximated PEs. Note that the error of the approximation again decreases as the system size increases. From a practitioner’s perspective, the performance of the asymptotic approximation under FIFO and RA-s means that one can obtain valid and sufficiently accurate results from studying a drastically simpler deterministic model.

To finalize this section, recall that one of the strong assumptions of our original (nonasymptotic) model for the rationing regimes under consideration is that the seller does not update the information about the remaining number of units Q_t during the sales horizon. However, in an asymptotic sense, consumers can infer the inventory level Q_t in real time; hence, our proposed fluid solution for these partial information settings is also an equilibrium fluid solution for the game in which the seller provides information about the inventory level over time. Therefore, our model can be viewed as an asymptotic first-order approximation to the full information game.

Figure 5 Asymptotic Approximation for the RA Rationing Rule for an Intermediate Supply Case with Time-Homogeneous Valuations Unif[0, 1], $T_s = w = 1$, $p_h = 0.5$, and $\rho = 0.7$



Q_0	Exact PE		Approx. PE		Error (%)
	Reservations	Buy-nows	Reservations	Buy-nows	
7	7.65	2.35	8.37	1.63	14.25
14	15.85	4.15	16.73	3.27	8.81
35	41.06	8.94	41.83	8.17	3.07
70	83.42	16.58	83.65	16.35	0.46
140	167.05	32.95	167.30	32.70	0.25

Note. The limiting probability of getting a reserved item is $c^\infty(H) = 0.64$.

6. Revenue Optimization

Following the usual approach for analyzing Stackelberg form games, we have assumed so far that the parameters that describe the business environment are fixed and studied how strategic consumers behave in equilibrium. Our primary focus in this section is the seller’s revenue optimization problem under the FIFO rationing rule; that is, we determine which parameters the leader has to announce. The optimal revenue performance under RA and under a fixed-price policy (FP) are our benchmark measures. Next, we study the value of the market composition information for the retailer, that is, we compute how the optimal revenue changes if a fraction of consumers behaves non-strategically (i.e., myopically). Finally, we study the consumer surplus achievable under FIFO, RA, and FP.

In what follows we assume that consumers’ valuations are time-homogeneous to parallelize the setting of Aviv and Pazgal (2008), and simplify the notation $F(\cdot, t)$ to $F(\cdot)$. Throughout all studies in this section we use the fluid equilibrium in the optimization problem, which is justified by the convergence results of §5 and the accuracy of the approximation. Furthermore, as mentioned earlier, this is the equilibrium under

a deterministic demand model with no integrality constraints.

The first step in formulating the seller’s problem is to scale back the parameters of our model (i.e., take the inverse of the transformation (2)), and write the optimization problem in terms of the original parameters. Defining α as the seller’s discount factor, the optimal revenue under FIFO is given by the solution to

$$\begin{aligned}
 V_{\text{FIFO}}(\bar{Q}) &= \max_{T, Q_0, p_l, p_h} \left\{ p_h \lambda \int_0^T e^{-\alpha t} \mathbb{1}\{Q_t > 0\} \bar{F}(H(t)) dt \right. \\
 &\quad \left. + p_l e^{-\alpha T} \min\{(Q_0 - \Lambda_{H_B}(T))^+, \Lambda_{H_R}(T)\}, \right. \\
 &\quad \left. \text{subject to } p_l \leq p_h, Q_0 \leq \bar{Q} \right\}. \quad (17)
 \end{aligned}$$

The optimal revenue under our RA benchmark is the solution to the maximization problem:

$$\begin{aligned}
 V_{\text{RA}}(\bar{Q}) &= \max_{T, Q_0, p_l, p_h} \left\{ p_h \lambda \int_0^{T_s} e^{-\alpha t} \mathbb{1}\{Q_t > 0\} \bar{F}(H(t)) dt \right. \\
 &\quad \left. + p_l e^{-\alpha T_s} \min\{(Q_0 - \Lambda_{H_B}^{\text{RA}}(T_s))^+, \Lambda_{H_R}^{\text{RA}}(T_s)\} \right. \\
 &\quad \left. + V_C, \text{ subject to } p_l \leq p_h, Q_0 \leq \bar{Q} \right\}, \quad (18)
 \end{aligned}$$

where V_C is the revenue collected during the clearance season, i.e.,

$$\begin{aligned}
 V_C &\triangleq \frac{1}{\alpha} p_l \bar{F}(p_l) \lambda (\exp(-\alpha T_s) - \exp(-\alpha \min\{T, \tau^*\})), \\
 &\quad \text{for } \alpha > 0, \quad (19)
 \end{aligned}$$

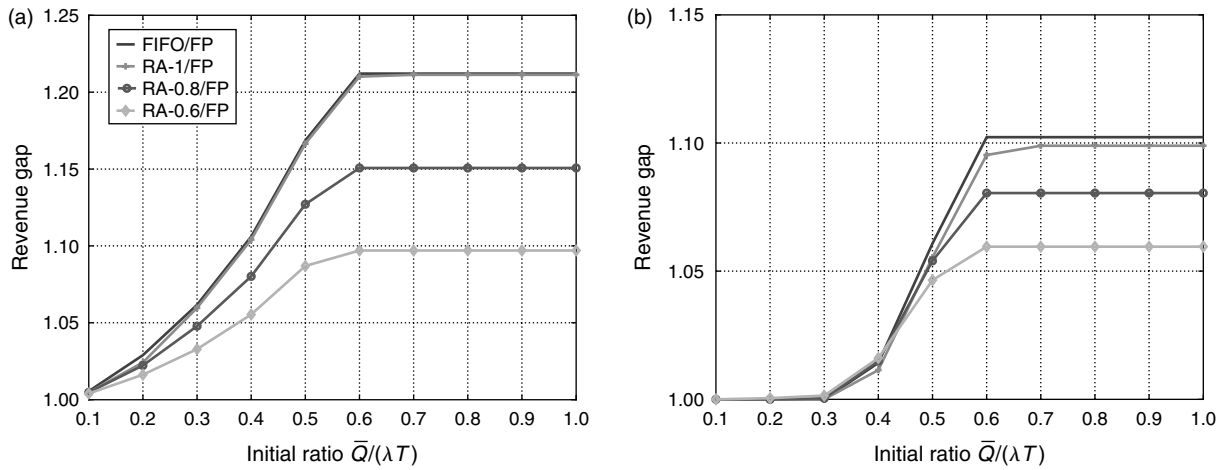
and where $\tau^* \triangleq T_s + (Q_0 - \lambda \bar{F}(p_l) T_s)^+ / (\lambda \bar{F}(p_l))$ stands for the inventory run-out time. Define s as the proportion of time during the sales season where the full price is used, i.e., $s \triangleq T_s / T$, and denote this rationing rule as RA- s . In particular, RA-1 denotes the extreme case where there is no clearing season and excess inventory is allocated randomly at time T . In case of multiple Bayesian-Nash equilibria, we use the Pareto-dominant one in our reports.

Unfortunately, no simple analytical solutions to (17) and (18) exist, and therefore we have to rely on numerical experiments to optimize the policy parameters. We present our results in §6.2 below, but first we present some general structural properties that hold in our case.

6.1. Structural Properties of Optimal Solutions

To simplify the calculations and gain some insights, we will assume in this subsection that the original distribution of valuations is $F \triangleq \text{Unif}[0, 1]$. Assume T

Figure 6 Revenue Increase of FIFO and RA- s , $s \in \{0.6, 0.8, 1\}$ with Respect to an Optimal Fixed-Price Policy Under Different Initial Availability Ratios $\bar{Q}/(\lambda T)$ for (a) $\alpha = 0.1$ and (b) $\alpha = 1$



Note. Default values of parameters are $\bar{Q} \leq 10$, $T = 1$, $\lambda = 10$, $w = 5$, and valuations $\text{Unif}[0, 1]$.

is fixed. Let V be the seller’s optimal revenue under either strict priorities (e.g., FIFO) or RA-1. Proposition A6 in the e-companion characterizes some structural properties of the optimal solutions. Our main findings regarding the optimal revenue V are as follows:

- When $\bar{Q}/(\lambda T)$ is relatively small and/or the seller’s discount for reservations αT is relatively large, the seller does not implement price discrimination; she puts up all the inventory for sale (i.e., $Q_0^* = \bar{Q}$), and charges a unique price $p_i^* = p_h^* \geq 1/2$.
- When $\bar{Q}/(\lambda T)$ is relatively big and/or the seller’s discount for reservations αT is relatively small, the seller has an incentive to achieve price discrimination by adding the reservation channel. In this case, the benefit of adding a reservation channel to a single fixed-price operation can increase revenues by as much as 33% (see §A1.4 in the e-companion for details).

6.2. Numerical Experiments

6.2.1. Revenue Performance. We consider an illustrative base case with $\bar{Q} \leq 10$, $\lambda = 10$, $w = 5$, and a valuation distribution $F \triangleq \text{Unif}[0, 1]$. Recall that \bar{Q} represents the seller’s endowment, and that she chooses $Q_0 \leq \bar{Q}$. Assume that the length of the selling horizon is set in advance, as it is generally the case for short life cycle products. Let $T = 1$ and compare the revenue gaps of FIFO and RA- s , for $s \in \{0.6, 0.8, 1\}$,¹¹ with respect to FP, under different initial availability scenarios $\bar{Q}/(\lambda T)$. Our main findings follow.

(i) **FIFO delivers higher revenue than RA- s .** Figure 6 plots the revenue gaps for $\alpha = 0.1$ and $\alpha = 1$. The setting mimics that of the dual auction and list price

channel (see Caldentey and Vulcano 2007, §5.2). We see that the FIFO regime earns up to 21.2% more than FP (when $\alpha = 0.1$, $s = 0.6$). The relative benefit of FIFO over RA- s is more pronounced when the clearance season is longer (i.e., when s is lower). For instance, FIFO is able to add more than 12% of revenues over RA-0.6.

Discount factors play a critical role for achieving price discrimination and extracting higher revenues. Indeed, the prevalence of FIFO and RA- s over FP occurs when $\alpha < w$, because the seller can take advantage of the consumers’ impatience. This is consistent with the findings of Aviv and Pazgal (2008, §7.3), and it is also aligned with traditional results of the economics literature.¹² When α is noticeably smaller than w , FIFO can deliver significantly more revenues than RA-0.6 and RA-0.8.

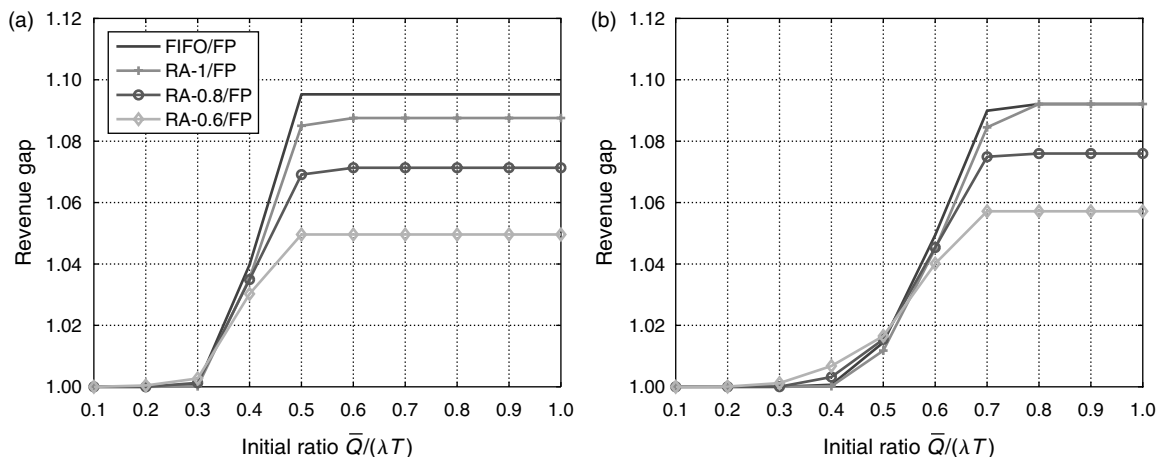
Another factor studied in Figure 7 is the heterogeneity of the consumers’ valuations. To this end, we assume that valuations follow a symmetric Beta(b, b) distribution¹³ with shape parameters $b = 0.5$ and $b = 2$. Relatively speaking, a seller applying FIFO can benefit more over the other selling mechanisms when facing highly to moderately heterogeneous consumers (i.e., valuations with higher variance described by a smaller value of the shape parameter b) under medium to high initial availability ratios. The optimal number of units put up for sale is moderate

¹² The classic work of Coase (1972) argues that a monopolist facing rational, patient customers who anticipate future lower prices is forced to price at the marginal cost and earn zero profit. Discount factors justify this assertion; if the seller is more patient than the customers, she may take advantage of price discrimination (see von der Fehr and Kuhn 1995).

¹³ The Beta(a, b) distribution has bounded support $[0, 1]$ and is symmetric when $a = b$. When $a = b = 1$, the Beta distribution becomes the $\text{Unif}[0, 1]$. Lower values of parameters a and b correspond to higher variance of the distribution.

¹¹ The choice of s is aligned with a recent empirical study by Nakamura and Steinsson (2008); see §4.2 for details.

Figure 7 Revenue Increase of FIFO and RA- s , $s \in \{0.6, 0.8, 1\}$ with Respect to an Optimal Fixed-Price Policy Under Different Initial Availability Ratios $\bar{Q}/(\lambda T)$ for Valuations Following a Beta(b, b) Distribution with (a) $b = 0.5$ and (b) $b = 2$



Note. Default values of parameters are $\bar{Q} \leq 10$, $T = 1$, $\lambda = 10$, $\alpha = 1$, and $w = 5$.

(i.e., $Q_0^*/(\lambda T) = 0.5$ for valuations Beta (0.5, 0.5), and $Q_0^*/(\lambda T) = 0.7$ for valuations Beta (2, 2)).

Note also that to achieve optimal revenues, FIFO needs less inventory when facing highly heterogeneous consumers.¹⁴

(ii) **The FIFO revenue is comparable with the auction-based mechanism.** The setting for the results presented in Figure 6 mimics that of the dual auction and list price channel (Caldentey and Vulcano 2007, §5.2). Interestingly, the revenue increases delivered by RA-1 and FIFO are comparable to those from a dual auction and list price channel, with FIFO weakly dominating RA-1. For $\alpha = 0.1$, Caldentey and Vulcano (2007) report 22% revenue increase from the dual auction list price channel, whereas FIFO delivers 21.2% increase. Therefore, we are (almost) achieving auction-based revenues with a time-based priority, list price mechanism like FIFO.

(iii) **FIFO discounts more aggressively than RA- s .** Figure 8 plots the optimal prices set under FIFO, RA-0.8, and RA-0.6 as a function of the initial inventory availability $\bar{Q}/(\lambda T)$. Figure 9 shows the number of units sold at each of the prices. Notice that for sufficiently small $\bar{Q}/(\lambda T)$, all three mechanisms operate in the limited supply regime, therefore, charge the same price p_h and offer zero markdown. This is the case for $\bar{Q}/(\lambda T) = 0.1$ in Figure 8. As expected, the optimal prices are decreasing functions of the initial supply, until the point when a mechanism enters the abundant supply regime. Once in the abundant regime, the optimal prices p_h, p_l , and the amount put up for sale Q_0 remain the same. For all initial availability ratios

¹⁴ The optimal supply-demand ratio is $\rho^* = Q_0^*/(\lambda T) = 0.5$ when selling to consumers with Beta(0.5, 0.5) valuations, as opposed to $\rho^* = 0.6$ when selling to consumers with Beta(1, 1) or $\rho^* = 0.7$ for Beta(2, 2) valuations.

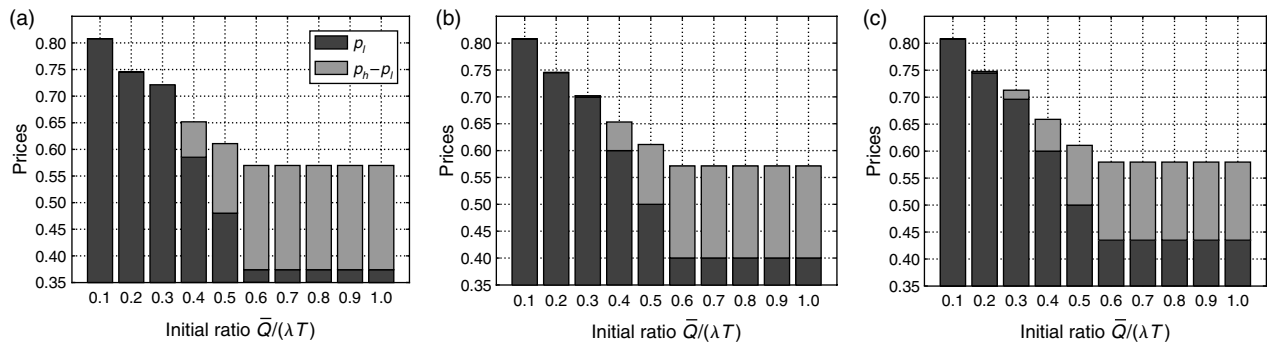
$\bar{Q}/(\lambda T)$, the price p_h is similar for FIFO and RA- s , but FIFO tends to discount more aggressively. In this way, more people are kept within the game and more units are sold at the full price because of the scarcity threat.¹⁵

(iv) **FIFO sells more items through the buy-now channel.** In terms of inventory availability, from Figure 9 the most beneficial cases in favor of FIFO occur when the ratio $\bar{Q}/(\lambda T)$ is moderate to large, so that there are opportunities for price discrimination (also exploited, though less effectively, by RA- s). Note that for $\bar{Q}/(\lambda T) \geq 0.6$, the optimal inventory level is $Q_0^* \leq 6$. That is, the seller benefits from deliberately introducing some scarcity in the market. When comparing the buy-nows of FIFO versus the buy-nows of RA-0.8 and RA-0.6, consumers are more eager to buy immediately under the former, even when constraining the comparison to the period during which RA- s exhibits the full price.

In summary, revenue-wise, the most beneficial cases in favor of FIFO over RA- s and FP occur when the seller is more patient than the consumers, and, in particular, when (1) the inventory availability $Q_0/(\lambda T)$ is moderate and/or (2) the dispersion of the consumers' valuations is moderate to large. In our experiments, the revenue advantage of FIFO with respect to the fixed-price policy can reach a level of up to 21%, and always (weakly) dominates the implementation of a clearance season. The key to FIFO's advantage is the ability to price discriminate (as opposed to FP) plus the asymmetry of the rationing rule (as opposed to RA- s).

¹⁵ For the sake of completeness, we include in the e-companion, §A3.1, the prices and split of units for RA-1. We observe that RA-1 needs to sell more units than FIFO in order to almost match revenues.

Figure 8 Prices p_l and Markdowns $= p_h - p_l$ Under Different initial Availability Ratios $\bar{Q}/(\lambda T)$ for (a) FIFO, (b) RA-0.8, and (c) RA-0.6



Notes. The total height of the bars is giving p_h . Default values of parameters are $\bar{Q} \leq 10$, $T = 1$, $\lambda = 10$, $\alpha = 1$, $w = 5$, and valuations $\text{Unif}[0,1]$.

6.2.2. Value of Market Composition Information.

Consider the *mixed-market* case, where there are two different types of consumers. Here, the total arrival rate λ is split between a fraction γ , $0 \leq \gamma \leq 1$, of myopic consumers, and a fraction $1 - \gamma$ of strategic consumers. We begin assuming that γ is common knowledge.

The myopic, impulsive consumers with valuation $v \geq p_l$ behave according to the simple strategy “buy now if own valuation is higher than p_h , and reserve otherwise.” The strategic ones choose the channel that maximizes their expected utility. Both types of consumers participate in the clearing of excess inventory at the end of the selling season. Section A3.2 of the e-companion provides the details of the analysis.

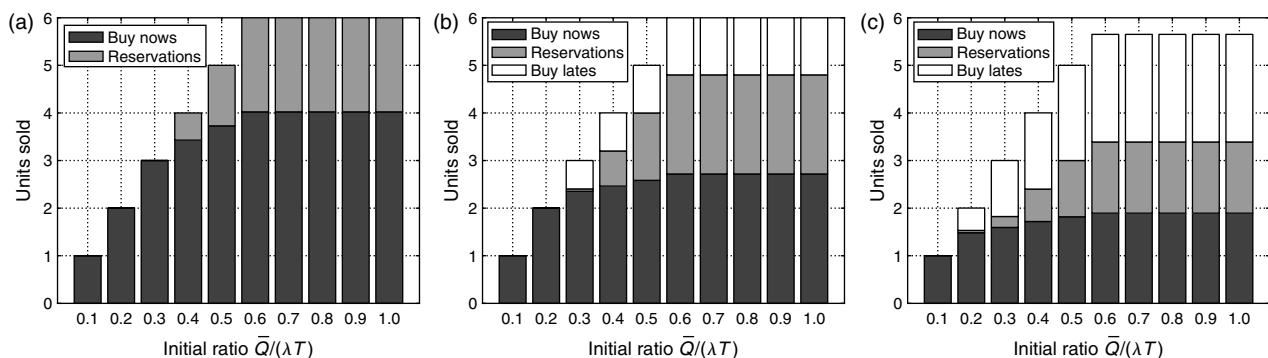
Interestingly, the stochastic, exact consumer strategy under FIFO rationing rule is sensitive to the parameter γ , but its asymptotic counterpart is invariant with respect to it. In other words, under the asymptotic FIFO regime, forward-looking consumers can ignore the fraction of myopic consumers in order to compute the (optimal) equilibrium strategy. This somewhat surprising result is anchored in the following two features: (1) Strategic consumers can assess the time of

the last marginal arrival who will get a unit (i.e., the value of τ^* defined in (11)), which does not depend on γ ; hence, they can also assess the limiting probability of getting a unit through each of the channels (which according to Theorem 2 is just 1 or 0). (2) Given the setting of the game (Q_0 , T , p_l , and p_h), the other factors that define consumers’ utility at the moment of making the purchasing decision are v_τ and τ , which do not depend on γ either. For the RA rule, the factor γ is included in both the exact and asymptotic strategies; the reason being that in this case, a consumer is uncertain about the fact of getting an item through the reservation channel in both regimes

(i) **FIFO delivers higher revenues in the mixed-market case.** Indeed, in Figure 10(a) we analyze the revenue of FIFO and RA-s with respect to FP under this mixed-market framework. We see that FIFO consistently achieves the best revenue performance, just matched by RA-1 when there is more than 20% of myopic consumers in the market.

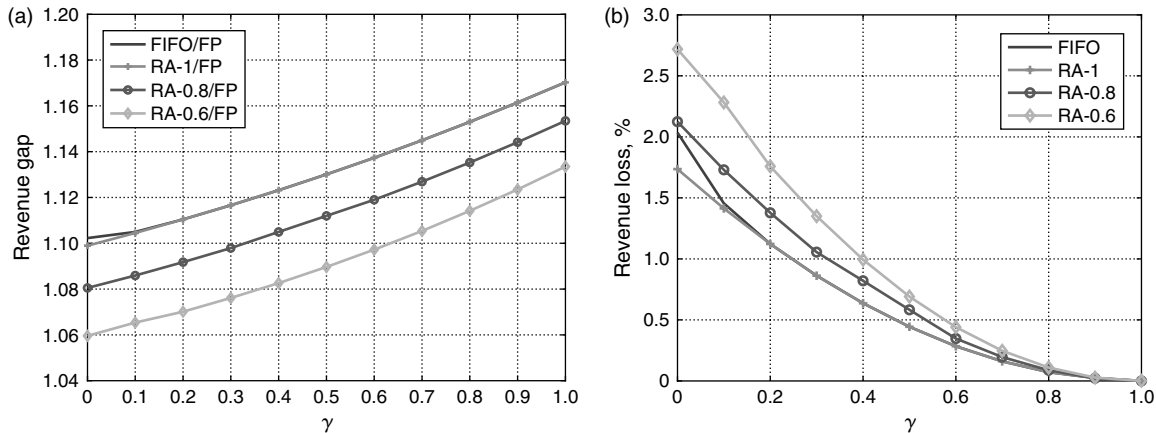
(ii) **Ignoring that the consumers are strategic can be costly.** We study the scenario when the seller incorrectly assumes that all consumers are myopic while in fact just a fraction γ of them indeed are. In Figure 10(b) we plot the revenue gap between

Figure 9 Split of Units Sold Under (a) FIFO, (b) RA-0.8, and (c) RA-0.6 Under Different Initial Availability Ratios $\bar{Q}/(\lambda T)$



Notes. The height of the bars stands for the total number of units sold during the selling horizon. Default values of parameters are $\bar{Q} \leq 10$, $T = 1$, $\lambda = 10$, $\alpha = 1$, $w = 5$, and valuations $\text{Unif}[0, 1]$. For RA-s, both reserved units and those *buy-lates* cleared during the last $T(1 - s)$ time units are sold at p_l .

Figure 10 Effect of Market Composition



Notes. (a) Revenue increases of FIFO and RA- s , $s \in \{0.6, 0.8, 1\}$ with respect to an optimal fixed-price policy for different fractions γ of myopic consumers. (b) Relative revenue loss when the seller optimizes as if all consumers were myopic when just a fraction γ of them are. Default values of parameters are $\bar{Q} = 10$, $T = 1$, $\lambda = 10$, $\alpha = 1$, $w = 5$, and valuations $\text{Unif}[0, 1]$.

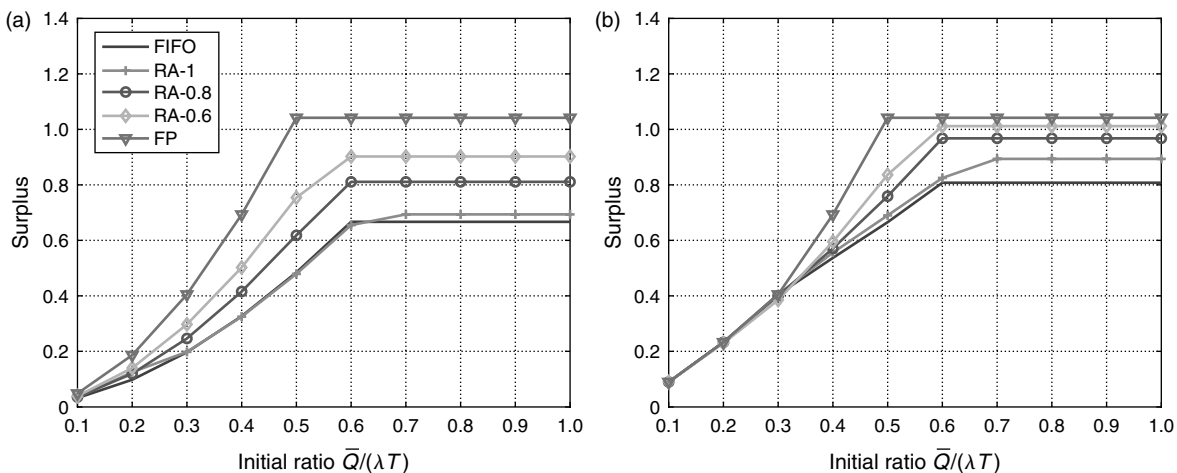
the seller optimizing Q_0 , p_l , and p_h under the correct proportion of myopic consumers (i.e., knowing the value of γ) and the seller optimizing under the wrong assumption that everybody is myopic. It is clear that the negative impact of this misbelief for all the rationing rules under consideration is decreasing in the proportion of myopic consumers. The four policies studied have a comparable sensitivity to market composition information, specially when more than half of the consumers are myopic. Ignoring market composition could decrease the revenue potential by more than 2%, and is even more substantial when consumers are more heterogeneous (e.g., when $F = \text{Beta}(0.5, 0.5)$, the suboptimality gap of FIFO could reach 3.2%). We also verify that this misbelief hurts FIFO less than RA- s .

6.2.3. Consumer Surplus. Figure 11 plots the total surplus obtained by consumers under FIFO,

RA-1, RA-0.8, RA-0.6, and FP (expressions are given in §A3.3 in the e-companion). Two observations are apparent:

(i) **FIFO is able to extract more consumer surplus than the other policies (up to 30% more than FP when $\alpha = 0.1$, and around 20% more than FP when $\alpha = 1$).** The surplus under RA- s , especially for $s = 0.6$ and 0.8 , is higher than under FIFO, because of three reasons: (1) the length of the period when the high price is charged is shorter; (2) consumers who placed reservations get items earlier, therefore incur less disutility from waiting; (3) a substantial number of consumers can get an item at a low price during the clearance season. The difference with respect to RA- s would have been less had we included a consumer search cost in the utility function for the RA- s case (recall that under the latter mechanism, consumers typically need to revisit the store).

Figure 11 Total Aggregated Consumer Surplus of FIFO, RA- s with $s \in \{0.6, 0.8, 1\}$, and FP Under Different Initial Availability Ratios $\bar{Q}/(\lambda T)$ for (a) $\alpha = 0.1$ and (b) $\alpha = 1$



Note. Default values of parameters are $\bar{Q} \leq 10$, $T = 1$, $\lambda = 10$, $w = 5$, and valuations $\text{Unif}[0, 1]$.

(ii) **Under some parameters, FIFO can deliver higher surplus than RA-1, as well as higher revenue.** For example, this is the case for $\alpha = 0.1$ and $0.4 \leq \bar{Q}/(\lambda T) \leq 0.6$, i.e., when the supply is intermediate (Figure 11(a)). The additional surplus delivered ranges between 0.49% and 1.83%. The surplus difference is even greater if consumers are more homogeneous (e.g., when $F = \text{Beta}(2, 2)$, it varies between 1.32% and 2.20%). The reason is that the high priority given by FIFO to early reservations is offset by their longer waiting time, which forces some customers to choose the buy-now option. Their losses due to paying a higher price are compensated by the utility of an immediate ownership. This suggests that for certain types of “hot” products (with high disutility of waiting), substituting a short clearance season by the mechanism with FIFO reservations can be a win-win solution in terms of both revenue and consumer surplus.¹⁶ In general though, FP retains the highest consumer surplus at the expense of lowest revenues.

7. Conclusions

In this paper, we develop a stylized model where a seller facing an arrival stream of strategic consumers operates a *selling with binding reservations* scheme. Upon arrival, each consumer, trying to maximize his own surplus, must decide either to purchase at a high price and get the item at no risk or to place a reservation at a discount price and wait until the end of the sales season when the leftover units are allocated according to first-in-first-out (FIFO) priority. As a first benchmark, we use the two-period, preannounced discount model studied by Aviv and Pazgal (2008), that we call random allocation (RA). Here, consumers arriving early in the sales horizon choose between buying now or waiting for the beginning of the clearance season, where leftover units are allocated randomly among the consumers who took the gamble. More consumers come during the clearance season, and deplete the remaining inventory (if any). The second benchmark is a fixed-price policy.

For both FIFO and RA settings, the seller announces the price path at the beginning of the horizon. Consumers know that price will decline over time, but they are also aware of the risk of later nonavailability. Their private information consists of the arrival time and the unit valuation. Of course, for consumers with valuation between the full price and the discount price, the optimal strategy is to always place a reservation. For the ones that may take the gamble,

using a time-sensitive utility function, we show that their purchasing equilibrium strategy is of the threshold type; that is, a consumer will place a reservation if and only if his own valuation is lower than a function of his arrival time.

At a theoretical level, we prove that a symmetric purchasing equilibrium always exists for a broad class of rationing rules that includes FIFO and RA, and we use a contraction algorithm in a function space to find it. The procedure is computationally intensive and provably convergent under specific conditions. To overcome these limitations, we develop an asymptotic, fluid-type approximation for the two settings, where the initial number of units and the demand rate grow proportionally large. This limiting regime can also be justified as a model with deterministic demand.

Because of the simplicity and accuracy of the asymptotic analysis, we solve the seller’s revenue optimization problem under this limiting regime. We observe that the FIFO rationing rule consistently achieves the best performance. A retailer can extract additional revenues if she is more patient than the consumers and if she can price discriminate among impulsive consumers who buy now and those ones who make the strategic decision to wait. Furthermore, the FIFO rule allows to discriminate deeper among the consumers who made to strategic decision to wait, enforcing a time-based asymmetry. Compared to RA, the most beneficial cases revenue-wise occur when (1) there is a moderate number of units put up for sale with respect to the expected demand and/or (2) the dispersion of the consumers’ valuations is moderate to high. The relative benefit of FIFO versus the standard markdown practice is even more emphasized when the clearance season is longer. Our numerical experiments show that the revenue gap can exceed 12%. Moreover, its advantage is still preserved when the market is composed by both myopic and strategic consumers.

In conclusion, the use of binding reservations under a FIFO rationing rule brings additional revenues for the seller and provides appealing benefits to the consumers. Given the narrow margins of retail operations, it can have a profound impact on these businesses’ profitability. Overall, we believe it is a selling mechanism that deserves further exploration to be implemented in practice.

8. Electronic Companion

An electronic companion to this paper is available as part of the online version that can be found at <http://mansci.journal.informs.org/>.

Acknowledgments

The authors are grateful to Yossi Aviv (Washington University in St. Louis), Martin Lariviere (Northwestern University), Candace Yano (University of California,

¹⁶ Elmaghraby et al. (2009, §3.5), observe a similar win-win result when the consumers’ sensitivity to markdown (i.e., the extra demand generated by the markdown) is low. The consumers are less sensitive to markdowns if the distribution of their valuations is more homogeneous. In this sense, our observation of the increased surplus when consumers are more homogeneous parallels the one of Elmaghraby et al. (2009).

Berkeley), Victor Araman (American University of Beirut, Lebanon), the associate editor, and three anonymous referees for their constructive feedback. This research was conducted while the first author was a Ph.D. student at the IOMS Department, Leonard N. Stern School of Business, New York University. During part of this research, the second author was visiting Universidad Torcuato di Tella in Buenos Aires, Argentina.

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