# Behavioral Anomalies in Consumer Wait-or-Buy Decisions and Their Implications for Markdown Management 

Manel Baucells (Virginia)<br>Nikolay Osadchiy (Emory)<br>Anton Ovchinnikov (Queen's)

## Markdowns in retail

- Markdowns are critically important for retailers:
- $1 / 3$ unit and $1 / 5$ dollar sales (Agrawal and Smith 2009)
- Markdown sales can significantly affect profitability:
- Net margin for a typical retailer is about 3\% (Damodaran 2015)
- Thus a $1 \%$ increase in revenue $\equiv \sim 33 \%$ increase in profit
- Markdowns attract low-end consumers, but some high-end consumers strategically wait for markdowns, eroding full-price sales.
- Understanding how consumers decide between buying at a full (tag) price or waiting for a markdown is critical for retailers to properly optimize markdowns.


## Wait-or-buy decision: a tradeoff between Money, Risk and Time



- A consumer with benefit of consumption $u$ sees an item with tag price $p<u$ and decides to buy now or wait until time $t$ for a markdown of $d$.
- If she buys now, then her surplus is $u-p$.
- If she waits then if the item is available, then her surplus is $u-p(1-d)$ realized after delay $t$ and a probability of the item being available of $q$. If item is not available, her surplus is zero.
- Should she buy now or wait?

Wait-or-buy decision is a multi-dimensional tradeoff between price/discount (money), likelihood of availability (risk) and delay (time).

## Benchmark model - discounted expected utility



- Effectively all papers, e.g., Besanko and Winston (MS1990), Aviv and Pazgal (MSOM2008), Liu and van Ryzin (MS2008) all 200+ citations, adopt the expected discounted utility (DEU) model:

$$
\begin{align*}
U(p, d, q, t) & =[u-p(1-d)] q e^{-\rho t}  \tag{1}\\
& \equiv \text { Money } \times \text { Ris } \times \text { Time }
\end{align*}
$$

- Hence, if

$$
\begin{equation*}
u-p>[u-p(1-d)] q e^{-\rho t} . \tag{2}
\end{equation*}
$$

then buy, otherwise wait.

- The DEU model assumes linearity and independence between dimensions


## How and why does the benchmark model fail?

People, however, perceive these tradeoffs as interdependent and non-linear (sensitivity to time depends on risk, etc.): in each dimension there are well-documented behavioral 'anomalies' which the DEU model cannot explain:

|  | Prospect A | v. | Prospect B | Response | N |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1. | ( 30 €, for sure, now) | v . | ( 40 €, with $80 \%$, now) | 58\% v. 42\% | 142 |
| 2. | ( $30 €$, with $10 \%$, now) | v. | (40 € , with 8\%, now) | 22\% v. 78\% | 65 |
| 3. | (100 fl, for sure, now) | v. | (110 fl, for sure, 4 weeks) | 82\% v. 18\% | 60 |
| 4. | (100 fl, for sure, 26 weeks) | v. | (110 fl, for sure, 30 weeks) | 37\% v. 63\% | 60 |
| 5. | ( $5 €$, for sure, 1 month) | v. | ( 5 €, with 90\%, now) | 43\% v. 57\% | 79 |
| 6. | (100 €, for sure, 1 month) | v . | (100 €, with $90 \%$, now) | 81\% v. 19\% | 79 |

Table: Rows 1-2 are taken from Baucells Heukamp (2010, Table 1). Rows 3-4 are taken from Table 1, Keren Roelofsma (1995, Table 1) (1 fl or Dutch Gulden in $1995=\$ 0.6$ ). Rows 5-6 from Baucells et al. (2009)

## Path forward

- Central concept of the paper: psychological distance between the prospects 'buy now at high price' (zero distance) and 'perhaps buy later at a low' (time, money and risk distance).
- When evaluating psychological distances people trade time, money and risk and such tradeoffs are nonlinear and prone to the behavioral anomalies we discussed
- We formulate a set of axioms that a preference relationship needs to satisfy in order to account for the three well-known behavioral anomalies
- Derive a general preference representation, that we call dPTT model
- Employ a parametric specification of the model
- Use experimental data to calibrate the model parameters
- Solve for the consumer wait or buy problem (simultaneous Nash game).
- Given the equilibrium, optimize markdowns and compare revenues (Stackelberg game).
- Validate recommended markdowns of dPTT vs. DEU out-of-sample.


## Discount, Probability, and Time tradeoff

- Each consumer desires at most one item.
- Let $x=\left(p_{x}, d_{x}, q_{x}, t_{x}\right) \in \mathcal{X}_{\tau}$, where
- $\tau$ - current calendar date
- $p_{x}-$ tag price
- $d_{x}$ - price discount
- $q_{x}$ - probability of the item being available
- $t_{x}$ - date of purchase $(\geq \tau)$, in case the item is available.
- Availability is revealed at time $t_{x}$.
- Notation: $\left(d, x_{-d}\right)$ or $\left(q, t, x_{-q t}\right)$ denotes $\left(p_{x}, d, q_{x}, t_{x}\right)$ or $\left(p_{x}, d_{x}, q, t\right)$, respectively.
- $\mathbf{0}=(0,0,0,0)$, no purchase.
- Let $\succeq_{\tau}$ be a preference ordering over pairs in $\mathcal{X}_{\tau}$ expressed at the current calendar date, $\tau$.
- For simplicity of exposition (and because the buyer an opt out), we only present preference conditions for $x, y \succeq \mathbf{0}$.


## Axioms

## A1

For all $\tau, \succeq_{\tau}$ is a complete and continuous ordering over $\mathcal{X}_{\tau}$.

## A2: Time invariance

$\forall x, y \in X, t_{x}, t_{y} \geq \tau$, and $\Delta \geq 0, x \sim_{\tau} y$ if and only if $\left(t_{x}+\Delta, x_{-t}\right) \sim_{\tau+\Delta}\left(t_{y}+\Delta, y_{-t}\right)$.
Given A2, we set $\tau=0$ and omit the subscript $\tau$.

## A3: Monotonicity

Let $\mathcal{X}^{0}=\left\{x \in \mathcal{X}: q_{x} e^{-t_{x}}=0\right\}$. For all $x \in \mathcal{X}$,
A3.0 if $x \in \mathcal{X}^{0}$, then $x \sim \mathbf{0}$.
A3.p let $p<p_{x}$. If $x \notin \mathcal{X}^{0}$, then $\left(p, x_{-p}\right) \succ x$.
A3.q let $q>q_{x}$. If $x \succ \mathbf{0}$, then $\left(q, x_{-q}\right) \succ x$.
A3.t let $t<t_{x}$. If $x \succ \mathbf{0}$, then $\left(t, x_{-t}\right) \succ x$.
A3.u there exist a $u \in(0, \infty)$ such that $(u, 0,1,0) \sim \mathbf{0}$.
By A2, the benefit of consumption $u$ is constant over time.

## A4:Effective price condition

For all $x, y \in \mathcal{X}$ such that $q_{x}=q_{y}$ and $t_{x}=t_{y}=0, x \succeq y$ if and only if $p_{x}\left(1-d_{x}\right) \leq p_{y}\left(1-d_{y}\right)$.

## Axioms

## A5: Probability and Time tradeoff

$\forall x \in \mathcal{X}, \theta, q \in[0,1]$ and $\Delta, t \geq 0$,

$$
\left(t_{x}+\Delta, x_{-t}\right) \sim\left(q_{x} \theta, x_{-q}\right) \text { if and only if }\left(t+\Delta, x_{-t}\right) \sim\left(q \theta, x_{-q}\right)
$$

The condition captures the psychologically intuitive notion that "time is intrinsically uncertain" (e.g., if a delay of 1 month is exchangeable with a probability factor of $80 \%$, then a delay of 2 months is exchangeable with a probability factor of $80 \% \cdot 80 \%=64 \%$, and this exchange holds independently of the base level of probability and time) The 'exchange rate' between probability and time $r(d)=\frac{1}{\Delta} \ln \frac{1}{\theta}$.

## A6: Price discount subendurance

$$
\begin{aligned}
& \forall x \in \mathcal{X}, \theta, q \in[0,1], \Delta, t \geq 0, d>d_{x} \\
& \quad \text { if } x \succ 0 \text { and }\left(t_{x}+\Delta, x_{-t}\right) \sim\left(q_{x} \theta, x_{-q}\right), \text { then }\left(p_{x}, d, q_{x}, t_{x}+\Delta\right) \succeq\left(p_{x}, d, q_{x} \theta, t_{x}\right)
\end{aligned}
$$

A6 captures pattern 5-6 (subendurance). Price discount, d, drives probabilistic patience. The assumption is also consistent with Kahneman and Tversky (2000)'s observation that individuals are willing to travel 10 minutes to grab a $33 \%$ discount on a calculator that costs $\$ 15$, but not willing to travel the same 10 minutes to grab a 5\% discount on a jacket that costs $\$ 100$.

## Axioms

The next conditions accounts for the 'common ratio' pattern 1-2.

## A7: SUB-PROPORTIONALITY

Let $x, y \in \mathcal{X}$ with $q_{x} \leq q_{y}$ and $t_{x}=t_{y}$. For all $\theta \in[0,1]$,

$$
\text { if } x \sim y \succ 0 \text { then }\left(\theta q_{y}, y_{-q}\right) \preceq\left(\theta q_{x}, x_{-q}\right)
$$

A5 and A7 imply sub-stationarity (the common difference pattern 3-4).

## A8: Restricted Probability-Price separability (hexagonal condition)

For all $x \in X$ with $t_{x}=0$ and $d_{x}=0, p, p^{\prime}, p^{\prime \prime} \geq 0, q, q^{\prime}, q^{\prime \prime} \in[0,1]$, if three of the following indifferences holds, the fourth one holds as well.

$$
\begin{array}{ll}
\left(p, q^{\prime}, x_{-p q}\right) \sim\left(p^{\prime}, q, x_{-p q}\right) & \left(p^{\prime}, q^{\prime}, x_{-p q}\right) \sim\left(p^{\prime \prime}, q, x_{-p q}\right) \\
\left(p, q^{\prime \prime}, x_{-p q}\right) \sim\left(p^{\prime}, q^{\prime}, x_{-p q}\right) & \left(p^{\prime}, q^{\prime \prime}, x_{-p q}\right) \sim\left(p^{\prime \prime}, q^{\prime}, x_{-p q}\right)
\end{array}
$$

## Representation

## Proposition dPTT

$\succeq_{\tau}$ on $\mathcal{X}_{\tau}$ satisfies A1-A8 if and only if, for some continuous functions: $v$, value function, strictly increasing with $v(0)=0$; a strictly increasing and concave psychological distance functions $s$, with $s(0)=0, s(1)=1$, and $s(\infty)=\infty$; and a decreasing probability discount rate $r(d)$,

$$
V_{\tau}(p, d, q, t)=v(u-p(1-d)) \cdot e^{-s(\sigma)}, u-p(1-d) \geq 0
$$

where $\sigma=\ln 1 / q+r(d)(t-\tau)$ is the psychological distance of the prospect.
(1) dPTT collapses into DEU if $v, s^{+}$, and $s^{-}$are the identity function and $r(d)$ is set constant.
(2) For immediate purchases, dPTT agrees with a prospect theory like formulation in which $v$ is a value function and $w(q)=e^{-s(-\ln q)}$ is a sub-proportional probability weighting function.
(3) For future purchases with no availability risk, dPTT agrees with a hyperbolic discounting model in which $f(t)=e^{-s(t)}$ is a sub-stationary time discount function.
(4) dPTT is time consistent: null purchases will be deemed indifferent both at $\tau=0$ and at any subsequent time $\tau>0$, favorable deal will remain favorable; and unfavorable deal will remain unfavorable.
(5) The term $s(\ln 1 / q+r(d) t)$ implies that risk and time distance are substitutes, and that individuals exhibit diminishing sensitivity to distance (of either type). In our context: because the option of waiting always exhibits time distance, individuals will not be very sensitive to reductions in $q$ (e.g., near 1 ).

## Parametric dPTT

## Parametric specification of dPTT

$$
\begin{array}{lr}
v(x)=x, x \geq 0, \text { and } v(x)=\kappa x, x<0, & \kappa \geq 1 \\
s(\sigma)=\sigma^{\beta} & 0<\beta \leq 1 \\
r(d)=\rho e^{\mu\left(d_{0}-d\right)}, & \rho>0, \mu \geq 0, d_{0} \in[0,1]
\end{array}
$$

Thus, given $(\beta, \rho, \mu) \in(0,1] \times(0, \infty) \times[0, \infty)$,

$$
V_{\tau}\left(p, d, q, t \mid \beta, d_{0}, \rho, \mu\right)=[u-p(1-d)] \cdot \exp \left\{-\left(\ln (1 / q)+\rho e^{\mu\left(d_{0}-d\right)}(t-\tau)\right)^{\beta}\right\}
$$

DEU is a special case of dPTT after setting $\beta=1$ and $\mu=0$ (or $r(d)=\rho$ ). Values of $\beta<1$ induce diminishing sensitivity to risk and time distance, $\ln 1 / q$ and $t$, respectively; values of $\mu>0$ induce more probabilistic patience when price discounts increase.

## Selling mechanism

- Two period model. Period 1: time 0; period 2: time $t$.
- Supply: $Q$ units
- Demand: $\lambda$, infitesemal consumers (fluid model, first order approximation to the stochastic model with Poisson demand, e.g., Maglaras and Meissner, 2006)
- Tag price: $p$
- Price discount: $d$ (available in period 2 )
- Probability of obtaining an item in period $i: q_{i}$
- Benefit of consumption: u, c.d.f. F, continuous
- Buy now if and only if:

$$
V_{0}(p, 0,1,0) \geq V_{0}\left(p, d, q_{2}, t\right)
$$

- Consumer problem: Which consumers buy now and which wait?
- Symmetric pure strategy threshold equilibrium: $\exists H$ s.t. consumers with $u>H$ buy now and others wait. Best response to $H, \mathcal{B}(\mathcal{H})$.


## Nash game: Consumer wait-or-buy strategy

## Proposition: Consumer Nash Equilibrium

- Abundant supply. If $Q \geq \lambda \bar{F}(p(1-d))$, then $q_{1}=q_{2}=1$, and the best response threshold is constant for all $H$ and given by

$$
\begin{equation*}
\mathcal{B}=\min \left\{p \cdot \frac{1-(1-d) e^{-s(r(d) t)}}{1-e^{-s(r(d) t)}}, 1\right\} \tag{3}
\end{equation*}
$$

There is a unique equilibrium given by $H^{*}=\mathcal{B}>p$.

- Intermediate supply. If $\lambda \bar{F}(p)<Q<\lambda \bar{F}(p(1-d))$, then $q_{1}=1, q_{2} \in(0,1)$, and the best response threshold is

$$
\begin{equation*}
\mathcal{B}(H)=\min \left\{p \cdot \frac{1-(1-d) e^{-s\left(\ln 1 / q_{2}+r(d) t\right)}}{1-e^{-s\left(\ln 1 / q_{2}+r(d) t\right)}}, 1\right\} . \tag{4}
\end{equation*}
$$

There is at least one equilibrium solving $\mathcal{B}\left(H^{*}\right)=H^{*}>p$.

- Limited supply. If $Q \leq \lambda \bar{F}(p)$, then $\mathcal{B}(H)=p$ on $H \in\left[p, F^{-1}\left(1-\frac{Q}{\lambda}\right)\right]$. We have that $H^{*}=p$ is always an equilibrium, but other equilibria with $H^{*}>p$ may exist.


## Proposition: Pareto-dominance

Let $H^{*}$ and $H^{* *}$ be two equilibria. If $H^{*}>H^{* *}$, then $H^{*}$ Pareto-dominates $H^{* *}$.

## Stackelberg game: Optimal markdown

Anticipating how consumers will decide, the firm (Stackelberg leader) decides on the optimal markdown to maximize the net present value from the tag- and markdown price sales (discounted at rate $\omega<\rho$ )

- The optimal markdown and discounted revenue under the dPTT model:

$$
\begin{aligned}
R^{d P T T}\left(d^{d P T T}\right)= & \max _{d}\left\{p \min \left(\lambda \bar{F}\left(H^{d P T T}\right), Q\right)\right. \\
& +e^{-\omega t} p(1-d) \min \left(\lambda\left(F\left(H^{d P T T}\right)-F(p(1-d))\right),\left(Q-\lambda \bar{F}\left(H^{d P T T}\right)\right)^{+}\right)
\end{aligned}
$$

- The optimal markdown under the DEU model:

$$
\begin{aligned}
R^{D E U}\left(d^{D E U}\right)= & \max _{d}\left\{p \min \left(\lambda \bar{F}\left(H^{D E U}\right), Q\right)\right. \\
& \left.+e^{-\omega t} p(1-d) \min \left(\lambda\left(F\left(H^{D E U}\right)-F(p(1-d))\right),\left(Q-\lambda \bar{F}\left(H^{D E U}\right)\right)^{+}\right)\right\} .
\end{aligned}
$$

The revenue gain from behavioral markdown management: $R^{d P T T}\left(d^{d P T T}\right)-R^{d P T T}\left(d^{D E U}\right)$.

## MAIN RESULT

## DPTT INCREASES MARKDOWNS AND INCREASES REVENUE

For given $\lambda, p, t$, and $u \sim U[0,1]$, let $Q>\lambda\left(1-\frac{1}{2} p\left(1+e^{\omega t-\rho t}\right)\right)$ so that $q_{2}=1$, $\sigma=r\left(d^{D E U}\right) t$ and the optimal markdown under DEU is $d^{D E U}=\frac{1}{2}\left(1-e^{\omega t-\rho t}\right)$. Then

$$
\left.\frac{\partial R^{d P T T}}{\partial d}\right|_{d=d^{D E U}}>0 \text { iff }-r^{\prime}\left(d^{D E U}\right) \times s^{\prime}(\sigma)<\frac{4\left(e^{s(\sigma)-\rho t}-1\right)\left(e^{s(\sigma)}-1\right)}{e^{s(\sigma)} t\left(1-e^{\omega t-\rho t}\right)\left(2-e^{-\omega t}-e^{-\rho t}\right)}
$$

- The condition is satisfied if $s(r(d) t) \geq \rho t$ (RHS positive) and either $\left|r^{\prime}\left(d^{D E U}\right)\right|$ or $s^{\prime}\left(r\left(d^{D E U}\right) t\right)$ small (LHS small).


## Optimal markdown, revenue, and revenue composition



Figure: (a) Revenue, (b) Equilibrium threshold $H$ and probability $q_{2}$, (c) Buy-now, buy-later, and total sold quantity as a function of discount $d$, under dPTT ( $\beta=0.9, \mu=1.95$ ) and DEU models. $Q_{0}=0.625, p_{h}=0.5, t=3, \lambda=1, u \sim \operatorname{Unif}[0,1], \omega=0.05, d_{0}=0.5$ and $\rho=0.13$.

Time unit is 3 weeks, aligned with the middle of the Bils and Klenow (2004) median price duration is 4.3 months or 18 weeks. $\rho=0.13$ is aligned with the time discount rate of $18 \%$ for payoffs between $€ 50$ and $€ 100$ and the wait of one month (Baucells et al. 2009); $\rho=0.13$ is also the best-fit DEU parameter for our data. $d_{0}=0.5$ is estimated through a pre-experiment. $Q=0.625$ ensures the intermediate or abundant supply.

## Intuition: WHY discounts are larger under dPTT?


(b)


Figure: Drivers of optimal markdown under dPTT. Panel (a) presents the base case $(\beta, \mu)=(0.9,1.95)$, and combinations $(1, \mu),(\beta, 0)$, and ( 1,0 ) (or DEU). Panel (b) presents the same for $(\beta, \mu)=(0.4,1)$.
Parameters: $Q=0.625, p=0.5, t=3, \lambda=1, u \sim \operatorname{Unif}[0,1], \omega=0.05, d_{0}=0.5$ and $\rho=0.13$.

- Marginal cost of increasing $d$ is smaller under dPTT: fewer consumers switch from the tag price to markdown sales. Hence, $d$ can be increased.
- Both subendurance and the decreasing sensitivity to psychological distance are important for the effect.


## Impact of behavioral effects on markdowns and revenue



Figure: (a) Optimal discount, (b) Revenue opportunity, (c) Revenue gain from incorporating the dPTT behavior as a function of $\beta$ and $\mu$. Parameters:
$Q_{0}=0.625, p_{h}=0.5, t=3, \lambda=1, u \sim \operatorname{Unif}[0,1], \omega=0.05, d_{0}=0.5$ and $\rho=0.13$.

## Optimal capacity decision and markdown timing




Figure: (a) Example of strategic rationing under dPTT: revenue (left axis) and tag price sales (right axis) as a function of markdown $d$, under dPTT ( $\beta=0.7, \mu=1.95$ ) and DEU models. Parameters: $Q=0.509, p=0.5, t=0.01, \lambda=1, u \sim U[0,1], \omega=0.05, d_{0}=0.5$, and $\rho=0.13$.
(b) Revenue under dPTT and DEU as a function of delay and optimal markdown. Parameters: $Q=0.625, p=0.5, \lambda=1, u \sim U[0,1], \omega=0.05, d_{0}=0.5$, and $\rho=0.13$.

- Capacity rationing can be optimal for a short selling season if behavioral anomalies are strong.
- dPTT sets markdowns that are larger, occur sooner, and generate more revenue.
- Analytical results supported by numerical studies.


## Robustness of dPTT results

| Scenario | Parameters | $d^{d P T T}$ | $d^{D E U}$ | $R^{\text {dPTT }}$ | $R^{d P T T}\left(d^{D E U}\right)$ | Rev. gain |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Expensive product | $p=0.75, Q=0.375$ | 17.3\% | 10.7\% | 0.2318 | 0.2234 | 3.76\% |
| Cheap product | $p=0.25, Q=0.825$ | 18.6\% | 10.7\% | 0.1925 | 0.1915 | 0.52\% |
| Short selling season | $t=1$ | 10.50\% | 3.8\% | 0.262 | 0.2568 | 2.02\% |
| Long selling season | $t=5$ | 22.8\% | 16.5\% | 0.2728 | 0.2708 | 0.74\% |
| Concentrated consumer valuations | $F=\operatorname{Beta}(4,4), Q=0.75$ | 18.0\% | 12.1\% | 0.2921 | 0.2871 | 1.74\% |
| Disperse consumer valuations | $F=\operatorname{Beta}(0.4,0.4)$ | 18.7\% | 10.5\% | 0.2609 | 0.2586 | 0.89\% |
| Low reference markdown | $d_{0}=0.25$ | 13.2\% | 10.7\% | 0.2626 | 0.2621 | 0.19\% |
| High reference markdown | $d_{0}=0.75$ | 24.5\% | 10.7\% | 0.2781 | 0.2683 | 3.65\% |
| Patient consumers | $\rho=0.06$ | 10.3\% | 1.5\% | 0.2589 | 0.2523 | 2.62\% |
| Impatient consumers | $\rho=0.2$ | 23.8\% | 18.1\% | 0.2771 | 0.2753 | 0.65\% |

Table: Impact of model parameters on markdowns, revenues and revenue gain from dPTT. The parameters are set to the baseline values, except those explicitly specified.

Sizable revenue gains (3.5\% or more) can be achieved by optimizing $d$ only.

## Pricing under dPTT: summary

- Observations:
- Revenue under dPTT is greater than under DEU.
- Optimal discount under dPTT is greater than under DEU.
- Shown analytically for a restricted parameter space.
- dPTT sells more units at tag price than DEU.
- dPTT sells more units in total than DEU.
- Sensitivity (for $(\beta, \mu) \in[0,1] \times[0,6])$
- dPTT delivers higher revenue as consumers deviate more from rationality.
- dPTT offers higher discounts without hurting full price sales:
- Decreasing sensitivity to psychological distance $(\beta<1)$ implies that those who buy early under DEU (zero distance) are very sensitive to additional distance, i.e., continue to buy now even when $d$ slightly increases.
- Decreasing probability discount rate $(\mu>0)$ makes consumers more patient.
- These two effects (caused by the anomalies we study) complement each other.


## Estimating model parameters

Recall:

$$
V_{\tau}\left(p, d, q, t \mid \beta, d_{0}, \rho, \mu\right)=[u-p(1-d)] \cdot \exp \left\{-\left(\ln \frac{1}{q}+\rho e^{\mu\left(d_{0}-d\right)}(t-\tau)\right)^{\beta}\right\}
$$

Where:

- $\rho$ is the base-line time discount factor - estimate from literature and our data
- $d_{0}$ is the reference percentage discount - estimate from pre-experiment
- $\mu$ is the subendurance parameter - estimate using an experiment
- $\beta$ is the sensitivity to psychological distance parameter - estimate using an experiment


## Estimating base-line time discount $\rho$ and reference percentage discount $d_{0}$

- For $\rho$ : (Baucells et al. 2009) found the time discount rate of $18 \%$ for payoffs $€ 50$ 100 and the wait of 1 month. Our $t=1$ is 3 weeks, hence $\rho=0.13 \approx 0.18 \times 3 / 4$. $\rho=0.13$ is also the best-fit DEU parameter for our data.
- For $d_{0}$ : we ran a pre-experiment with $N=32$ Canadian undergraduates:

Think about an end-of-season sale (markdown) at a retail store-such as Boxing day, for example. What is the percentage price discount that first comes to mind? [Free entry box]


Figure: Histogram of reference discount responses, $N=32$, mean $=51$, mode=median $=50$.

We therefore use $d_{0}=0.5$ in the estimation.

## Estimating $\beta, \mu$ : Experimental design

- Choice lists (Holt and Laury, 2002), followed by binary choice questions, each in a random sequence.
- Benefit of consumption: $u=\$ 250$; Tag price: $p=\$ 200$; Delay if wait: 3 weeks.
- Discounts: $d=5 \%, 15 \%, 25 \%, 50 \%, 75 \%$.
- Inventory liquidation recovery rate $27 \pm 5$ cents per $\$ 1$ (Elmaghraby et al. 2014)
- Probabilities: $q=10 \%, 20 \%, \ldots, 90 \%$.
- PrInce scheme (Johnson et al. 2014, Peter Wakker's group):
- Scenarios $(d, q)$ distributed in a physical/tangible form (sealed envelopes).
- 2 subjects will be selected and their scenario played.
- Subject's response used to play the scenario.
- Responses are framed as 'instructions' for the experimenters for how to play the scenario.


## Prior Incentive Scheme



## Implementation

Suppose that you went to a retail store and saw a product that you know you can resell for $\$ 250$ at any time. The product was priced at $\$ 200$ (two hundred dollars), so you picked the product from the shelf and were about to purchase. However, then you started thinking that in three weeks from today this product may be marked down. Thus the question was: should you buy the product now or wait for the markdown?

Recall that if you buy the product, you can re-sell it for $\$ 250$
The product is currently priced at $\$ 200$. The store has announced that in three weeks the product will be marked down by $25 \%$. The problem, however, is that it may be sold-out by then. From your experience. the likelihood that the product will be available could vary between $10 \%$ and $90 \%$.

Considering different possible likelinoods of the product being avallable, would you buy now or walt tor the markdown?

|  | Buy now | Wall for the markown |
| :---: | :---: | :---: |
| If there is a 10\% chance that the product will be available at the markdown | 0 | C |
| If there is a 20\% chance that the product will be avalable at the markdown | Q | 0 |
| If there is a 30\% chance that the product will be available al the markcown | 0 | 0 |
| If there is a $40 \%$ chance that the product will be available at the markdown | © | 0 |
| If there is a $\mathbf{5 0 \%}$ chance that the product will be available at the markdown | (1) | ( |
| If there is a $60 \%$ chance that the product will be available at the markdown | e | 0 |
| If there is a $\mathbf{7 0 \%}$ chance that the product will be avallabie at the markdown | Q | © |
| If there is a $80 \%$ chance that the product will be available ai the markcown | O | 6 |
| If there is a $90 \%$ chance that the product will be avallable at the markdown | 0 | 0 |

Recall that if you buy the product, you can re-sell it for $\$ 250$
The product is currently priced at $\$ 200$. The store has announced that the product will be marked down by $25 \%$ and there is a $50 \%$ chance that it will be avallable in three weeks.

Would you buy now or wait for the markdown?

Figure: Screenshots for the choice-list, and binary choice questions.

## Implementation, cont

$\mathrm{N}=64$, Emory undergrads
Two 'winners' were selected:

- ID49 (male): $d=25 \%, q=50 \%$, choice ='buy now'. Received $\$ 5$ for participation $+u-p=250-200=\$ 50$ and happily left.
- ID 11 (female): $d=25 \%, q=80 \%$, choice ='wait'. Received $\$ 5$ for participation and was asked to come back again in 3 weeks to learn availability.


## ID11



Scenario: $d=25 \%, q=80 \%$. Decision: wait 3 weeks. Outcome: product was available, payoff $=u-p(1-d)=250-200 \times(1-0.25)=\$ 100$.

## Indifference points

- Each choice list gives 9 ordered choices on $(d, q)$
- 62 / 64 subjects have a single switching point between buy and wait
- For choice list $i$, subject $j$, and discount $d_{i j}$, define indifference point $\tilde{q}_{i j}$ as a midpoint between highest $q$-'buy' and lowest $q$-'wait' ( $75 \%$ in the example on the right)
- Responses in choice lists are remarkably consistent with binary choices (67\% consistent in all choices, $23 \%$ in all but one)

ID11, $d_{i j}=25 \%$

| $q_{i j}$ | Buy-now | Wait |
| ---: | :---: | :---: |
| $10 \%$ | 1 | 0 |
| $20 \%$ | 1 | 0 |
| $30 \%$ | 1 | 0 |
| $40 \%$ | 1 | 0 |
| $50 \%$ | 1 | 0 |
| $60 \%$ | 1 | 0 |
| $70 \%$ | 1 | 0 |
| $80 \%$ | 0 | 1 |
| $90 \%$ | 0 | 1 |

An 'indifference point' implies that there exists $q(d)$ such that:

$$
\begin{equation*}
u-p=(u-p(1-d)) \exp \left[-\left(\ln \frac{1}{q(d)}+\rho e^{\mu\left(d_{0}-d\right)} t\right)^{\beta}\right] \tag{5}
\end{equation*}
$$

## Estimation approach

Solving for $q(d)$ we obtain: $q(d)=\exp \left\{-\left(-\ln \left(\frac{u-p}{u-p(1-d)}\right)\right)^{\frac{1}{\beta}}+\rho e^{\mu\left(d_{0}-d\right)} t\right\}$.


Figure: Observed, $\tilde{q}_{i j}$ and implied $q\left(\tilde{d}_{i j}\right)$ indifference probabilities.

Censored errors, fit LAD regression (Powell 1984): $\min _{\beta, \mu}\left[\sum_{i, j}\left|\tilde{q}_{i j}-q\left(\tilde{d}_{i j}\right)\right|\right]$.

## Estimates for $\beta$ and $\mu$



Figure: Estimates of consumer utility parameters $\beta$ and $\mu$.

Pooled estimates: $\beta=0.9, \mu=1.95$, standard errors obtained via bootstrapping/jackknife, Efron (1979).

## Markdowns with the estimated parameters



Figure: Experimental results and optimal markdowns: (a) Optimal discount, (b) Optimal revenue, (c) Revenue gain from incorporating the dPTT behavior as a function of $\beta$ and $\mu$. $x$ - individual estimates, $\square$ - individual median, $\boldsymbol{\Delta}$ - pooled estimate. Parameters of the pricing model: $Q=0.625, p_{h}=0.5, t=3, \lambda=1, u \sim \operatorname{Unif}[0,1], \omega=0.05$, $d_{0}=0.5$ and $\rho=0.13$.

## Parameter mis-estimation




Figure: Impact of unknown true parameters. The estimated values: $\beta=0.9, \mu=1.95$.
The baseline parameters: $Q=0.625, p=0.5, t=3, \lambda=1, u \sim \operatorname{Unif}[0,1], \omega=0.05, d_{0}=0.5$ and $\rho=0.13$.

- Losses from mis-estimation are $\approx 10 \times$ smaller than gains over DEU pricing.


## External validation of the dPTT results

- Our model predicts that if the firm offers the dPTT optimal discount of $18.7 \%$, it will make $\sim 1.5 \%$ more revenue than if it offers a DEU optimal discount of $10.9 \%$
- So lets just experimentally test this prediction!
- We run an Amazon Mechanical Turk survey.
- $n=600$ subjects, randomly endowed with $u=51, \ldots 70$, decide to buy at $p=50$ or wait 9 weeks for $p(1-d), d \in\{10 \%, 20 \%\} . q=100 \%$ in both cases, as per the model prediction.
- For $u=71, \ldots 100$ we assume all buy now (to minimize noise)
- For $u=50,49, \ldots 45 / 40$ we assume all will non-strategically wait
- For $u<45 / 40$ we assume no purchase.
- A complete set of $u=0, \ldots 100$ is defined as a "market." Subjects are assigned to markets in a first-come, first-served manner. Data contains 14 complete markets; for each we count the total number of waits and buys, and obtain revenue.
- Result: $R_{d=20 \%}=2,648>2,601=R_{d=10 \%}, p<0.01$. Relative improvement $\approx 1.8 \%$. Positive improvement in 13 out of 14 markets.
- Out-of-sample test strongly supports our prediction.


## Conclusions

- We address a fundamental question: how consumers make wait or buy decisions
- Wait or buy decisions are prone to multi-dimensional behavioral anomalies
- We develop a preference relation that accounts for observed behavioral anomalies:
- Formulate sufficient conditions (axioms) for the representation to hold
- Present a parametric 'example' (behavioral model)
- Solve for the wait or buy consumer problem/equilibrium
- Given the solution, optimize markdowns, compare revenues:
- Behavioral anomalies allow to offer higher discounts without hurting full price sales, but increasing total sales (market size), hence increasing revenues
- Use experimental data to calibrate the model and validate it out-of-sample
- We observe a substantial deviation from DEU
- Correctly accounting for this deviation leads to revenue gain of $1.5-2 \%$ ㄹ a $50-67 \%$ increase for a typical retailer's profit
- Results hold for a wide range of behavioral parameters and are robust wrp to errors in their estimation


## One sentence summary of The Paper:

Because of the way people feel about tradeoffs between discounts, risk, and time, firms should offer larger markdowns than the current models suggest, and by doing so obtain higher revenues.

## Thank you!

Comments/questions: nikolay.osadchiy@emory.edu

